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# THE SPIN WAVE SYSTEM IN FERROMAGNETIC LATTICS

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Recently, the Heisenberg system:

$$S_t = S \times S_{xx} + S \times h$$

has aroused the more and more interest. There are many works upon it in various places, such as soliton solutions, infinit conservation laws, etc. [1-7]. But this system is only the primary approximation of its original mathematical-physical model.

In ferromagnetic lattics[1, 2], spin vectors satisfies the relation:

$$\frac{\bar{h}}{2}\frac{\partial S_i}{\partial t} = \sum_k \frac{A}{2}[S_i,\ S_k] + [S_i,\ h]$$

where $i$ points to the $i$ th atom. The summing indices $k$'s point to these atoms which near to the $i$ th. $A$ is the exchange integral. $\bar{h}$ is the Planck constant. $h$ is a constant vector. For $\alpha$-Fe with ferromagnetic property is a simple cubic lattic with lattic constant $\bar{a}$, to which we have expansion:

$$\frac{A}{2}\sum_k S_k = \sum_{m=0}^{\infty} \tilde{\Delta}^m S_i$$

where $\tilde{\Delta}^m = \sum_{|\alpha|=m} a_\alpha D_x^{2\alpha}$, $(m = 1, 2, \ldots)$ are elliptic operators. $\alpha$'s are $N$- tuple indices. $a_\alpha$'s are positive constants depending on $\bar{a}$. The unequal relation between $\alpha$'s and the meaning of $\begin{pmatrix}\alpha' \\ \alpha''\end{pmatrix}$ are understood as usual.

In this paper, we will discuss the existence of the global weak solution of the system which is much nearer to the original model than before, only neglect the terms whose orders are higher than $2M$:

$$Z_t = Z \times \sum_{m=1}^{M} \tilde{\Delta}^m Z + f(x,\ t)\, Z + B(x,\ t) \qquad \text{in} \quad Q_T = \Omega \times [0,\ T) \qquad (0.1)$$

with the boundary-initial conditions:

$$\left.\frac{d^l Z}{d\gamma^l}\right|_{\partial\Omega} = 0 \qquad l = 0,\ 1,\ \ldots,\ M-1 \qquad \text{on} \quad S_T = \partial\Omega \times [0,\ T) \qquad (0.2)$$

$$Z(x,\ 0) = Z_0(x) \qquad \text{on} \quad \Omega \qquad (0.3)$$

where $Z(x,t)$, $Z_0(x)$, and $B(x,t)$ are all 3-dimensional vector functions. $f(x,t)$ is a $3\times 3$ matrix function. $\gamma$ is the out-normal vector. $x\in R^N$, $N<2M$. $\Omega$ is a $C^M$ bound domain in $N$-dimensional space. The $Z_0$ also satisfies the condition of compatibility: $\left.\frac{d^l Z_0}{d\gamma^l}\right|_{\partial\Omega}=0,\ l=0,1,\dots,M-1$.

In our model, $N$ is a finite number (generally it is 3). But $M$ can be arbitrary great. so the condition $N<2M$ can be satisfied easily, and it is quite natural.

The coefficient matrix of the highest order elliptic operator of the system (0. 1) is:

$$A_{2M}(Z)=\begin{pmatrix} 0 & -Z_3 & Z_2 \\ Z_3 & 0 & -Z_1 \\ -Z_2 & Z_1 & 0 \end{pmatrix}$$

It is a null definite matrix [3]. So (0. 1) is a high order degenerate quasilinear parabolic system.

Being based on the work of the second order ferromagnetic chain system by Zhou Yulin and Guo Boling [3, 4], we first discuss the solution of a system with a small parameter $\varepsilon\,(>0)$ ($a_0$ below will be determined later):

$$Z_t=\varepsilon(-1)^{M+1}\Big(\sum_{m=1}^{M}\Delta^m Z+a_0 Z\Big)+Z\times\sum_{m=1}^{M}\Delta^m Z+f(x,t)Z+B(x,t) \qquad (0.\ 4)$$

under the boundary-initial conditions (0. 2), (0. 3). And at last, we study the weak convergence of it when $\varepsilon$ tends to zero.

## 1

As preparing work, in this section, we will discuss a linear high order parabolic system:

$$u_t=(-1)^{M+1}A_{2M}(x,t)\tilde{\Delta}^m u+\sum_{|\alpha|\leqslant 2M-1}A_\alpha D_x^\alpha u+B(x,t) \qquad \text{in } Q_T \qquad (1.\ 1)$$

with the first boundary-initial conditions:

$$\left.\frac{d^l u}{d\gamma^l}\right|_{\partial\Omega}=0 \qquad l=0,1,\dots,M-1 \qquad \text{in } S_T \qquad (1.\ 2)$$

$$u(x,0)=u_0(x) \qquad \text{in } \Omega \qquad (1.\ 3)$$

where $u$, $u_0$ and $B$ are all $J$-dimensional vector functions. $x\in R^N$, $A_\alpha(x,t)=(A_\alpha^{ij}(x,t))_{ij=1}^{J}$, $|\alpha|=0,1,\dots,2M$. (When $|\alpha|=2M$, let $(A_\alpha)=A_{2M}$. It is same to $a_M$ in the following.) And $\left.\frac{d^l u_0}{d\gamma^l}\right|_{\partial\Omega}=0 \quad l=0,1,\dots,M-1$.

Denote by $\mathscr{B}$ a function space. Say $u=(u_1,\dots,u_J)\in\mathscr{B}$, if $u_i\in\mathscr{B}$, $i=1,2,\dots,J$. And $\|u\|_{\mathscr{B}}=\{\sum_{i=1}^{J}\|u_i\|_{\mathscr{B}}^2\}^{\frac{1}{2}}$.