J. Partial Differential Equations
Vol. 2, *No.* 2 (1989), 1—25

THE INITIAL VALUE PROBLEM FOR ONE CLASS OF THE SYSTEM OF PEKAR-CHOQUARD TYPE NONLINEAR SCHRÖDINGER EQUATIONS IN 3-DIMENSIONS①

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(Received December 22, 1986; revised September 20, 1988)

1. Introduction

H. J. Efinger [1] proposed the following nonlinear Schrödinger equations with nonlocal interaction:

$$iu_t + \Delta u + \int_{R^3} V(x-y)\,|u(y,t)|^2 dy\, u(x,t) = 0, \qquad \mathbf{R}_t \times \mathbf{R}^3 \tag{1.1}$$

where $u(x,t)$ is an unknown complex function, Δ is the Laplace operator and $V(x)$ is a suitable integral kernel defined in R^3. Equation (1.1) can be looked upon as the classical limit, in the sense of Hartree type approximations [2], of the field equation which describes a quantum mechanical nonrelativistic particle system interacting through a nonlocal potential $U(x)$: in the present case the potential is determined *self consistently* by

$$U(x) = \int |u(y,t)|^2 V(x-y)\, dy$$

Such nonlocal interactions arise in different areas of physics: a time dependent version of a meson-field-nucleon theory[3] and gravitational field-particle theory[4] with the inclusion of recoil effects leads to (1.1). A Hartree-Fock theory for a one-component plasma[5] provides a further example.

The Hartree-Fock equation and Pekar-Choquard type equations have been studied in many works [1-10]. Dirac also obtained Hartree-Fock equation with N ($N \geq 2$) body interactions in [11, 12]. N. D. Land, S. K. Ghosh and B. M. Deb have proposed Hartree-Fock-Choquard nonlinear Schrödinger equations with density-functional correct term in [13, 14].

In this paper we consider the following initial value problem of the system of Choquard type nonlinear Schrödinger equations with density-functional correct term in 3

①Research work supported by the Science Fund of the Chinese Academy of Sciences.

dimensions

$$iu_t - \Delta \vec{u} + K(\vec{u}(t)) + \beta q(|\vec{u}|^2)\vec{u} = 0, \ x \in \mathbf{R}^3, \ t \in R_+ \tag{1.2}$$

$$\vec{u}|_{t=0} = \vec{u}_0(x), \ x \in R^3 \tag{1.3}$$

where $\vec{u}(x, t) = (u_1(x, t), \ldots, u_n(x, t))$ is a N dimensional unknown complex functional vector, $i = \sqrt{-1}$, β is a real constant, $q(s)$ $(s \in [0, \infty))$ is a real valued function,

$$K(\vec{u}(t)) = \int_{R^3} V(x-y) U(x, y, t) \vec{u}(y, t) dy \tag{1.4}$$

$U(x, y, t) = (U_{jk}(x, y, t))$ is a $N \times N$ complex valued functional matrix,

$$U_{jk}(x, y, t) = u_j(x, t) u_k(y, t) - u_k(x, t) u_j(y, t), \ j, k = 1, 2, \ldots, N$$

Let $\overline{\vec{u}(y, t)}$ denotes the complex conjugation of $\vec{u}(y, t)$, real function $V(x)$ is a suitable integral kernel defined in R^3, and $V(x) = V(-x)$. The system (1.3) can be also written as following componentwise

$$iu_{mt} - \Delta u_m + K_m(\vec{u}(t)) + \beta q(|\vec{u}|^2) u_m = 0, \ m = 1, 2, \ldots, N \tag{1.5}$$

where

$$K_m(\vec{u}(t)) = \int_{R^3} \sum_{k=1}^{N} (u_m(x, t) u_k(y, t) - u_k(x, t) u_m(y, t)) V(x-y) \overline{u_k(y, t)} dy$$

$$= \int_{R^3} u_m(x, t) |\vec{u}(y, t)|^2 V(x-y) dy - \sum_{k=1}^{N} \int_{R^3} u_k(x, t) u_m(y, t) \overline{u_k(y, t)} V(x-y) dy$$

We also suppose the solution of the problem (1.2) (1.3) decays fast enough when $|x| \to \infty$.

This paper is organized as follows:

In sect. 2 the a priori estimations for the solution of problem (1.2) (1.3) are obtained. In sect. 3 first the local existence of the solution of problem (1.2) (1.3) is presented by using fixed point principle, and then the existence and uniqueness of the global smooth solution of problem (1.2) (1.3) are constructed. In sect. 4 we study the asymptotic properties for the solution of one class of Choquard nonlinear Schrödinger equation as $t \to \infty$ and gives the sufficient conditions of "blowing up" for the solution of problem (1.2) (1.3).

2. The a Priori Estimations

Lemma 1 *If* $\vec{u}_0(x) \in L_2(R^3)$, *then for the solution of problem* (1.2) (1.3) *the following integral equality holds*

$$\| \vec{u}(\cdot, t) \|^2_{L_2(R^3)} = \| \vec{u}_0(x) \|^2_{L_2(R^3)} \tag{2.1}$$

Proof The lemma is proved by multiplying (1.5) by $\overline{u}_m$ and integrating the