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AN EVOLUTIONARY CONTINUOUS CASTING PROBLEM OF TWO PHASES AND ITS PERIODIC BEHAVIOUR①

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Abstract

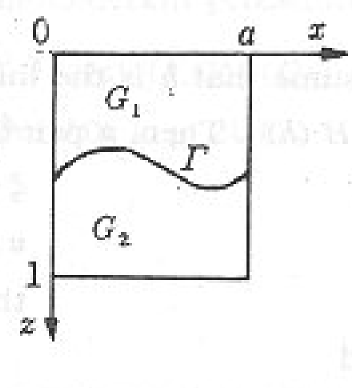
The present paper studies a continuous casting problem of two phases:

$$\frac{\partial H(u)}{\partial t}+b(t)\frac{\partial H(u)}{\partial x}-\Delta u=0 \quad \text{in} \quad \mathscr{D}'(Q_T)$$

where u is the temperature, $H(u)$ is a maximal monotonic graph, $Q_T=G\times(0,T)$, where $G=(0,a)\times(0,1)$ stands for the ingot. We obtain the existence and the uniqueness of weak solution and the existence of periodic solution for the first boundary problem.

1. Introduction, Definition of Weak Solution

There has been much work on the steady-state solution of the continuous casting problem (see [1] and [2]). Using the variational inequality method, Rodrigues studied the one-phase time dependent problem (see [3]), In this paper, the two-phase nonsteady state problem is discussed, and the existence and the uniqueeness of its weak solution are obtained under the conditionn that $b(t)$, the velocity of the ingot, is non-negative. Also, the existence of its periodic solution is studied, Some of the methods used in [4] and [5] are used here, and under weaker conditions, we get better results.



Assume that $G=(0,a)\times(0,1)$ denotes the ingot,

$$\partial_1 G=\partial G_1/\Gamma, \quad \partial_2 G=\partial G_2/\Gamma$$

Define:

$$\nabla=(\partial x,\partial z),\ Q_T=G\times(0,T)$$

Let the boundary surface of the solid and the liquid be

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$$\Gamma(t) = \{(x, z) \in G; \Phi(x, z, t) = 0\}, \qquad \Phi \in C^1(\overline{\Omega}_T)$$

Let $u = u(x, z, t)$ stand for the dimensionless temperature of the ingot, and assume $u \geq 0$ in liquid and ≤ 0 in solid. Let the velocity of the ingot be $\vec{v} = b(t)\vec{z}$.

Looking for the nonsteady state solution of two-phase problem means finding a pair (u_1, u_2, Φ), such that:

$$\alpha_i \frac{\partial u_i}{\partial t} + \alpha_i b(t) \frac{\partial u_i}{\partial z} - \Delta u_i = 0, \qquad (x, z) \in G_i(t), \ 0 < t < T \tag{1.1}$$

$$u_i = g_i, \qquad (x, z) \in \partial_i G, \ 0 < t < T \tag{1.2}$$

$$u_i = h_i, \qquad \text{on} \quad G_i(0) \tag{1.3}$$

$$u_i = 0, \qquad (x, z) \in \Gamma(t), \ 0 < t < T \tag{1.4}$$

$$\left[\frac{\partial u_i \partial \Phi}{\partial x \partial x} + \frac{\partial u_i \partial \Phi}{\partial z \partial z}\right]_{i=2}^{i=1} = \alpha[b(t)\Phi_z + \Phi_t] \tag{1.5}$$

where α_1, α_2 and α are positive constants; $\Gamma(t)$ is a curve on $G(t) = G \times \{t\}$; $G_i(t)$ is a domain bounded by $\Gamma(t)$ and $\partial_i G(t) = \partial_i G \times \{t\}$; $\Phi(x, z, t)$ is a C^1 function in $\overline{\Omega}_T$ such that

$$\Gamma(t) = \{(x, z, t) \in \overline{\Omega}_T; \ \Phi(x, z, t) = 0\}$$

$$(\Phi_t, \Phi_x, \Phi_z) \neq \vec{0} \quad \text{on} \quad \Gamma(t)$$

$$\Phi(x, z, t) < 0 \quad \text{in} \quad G_1(t); \ \Phi(x, z, t) > 0 \quad \text{in} \quad G_2(t)$$

The functions h_i and g_i are the initial and boundary data; and $S = \bigcup_{0 \leq t \leq T} \Gamma(t)$ is the "free boundary".

The above fomulations can be obtained either by establishing mathematical model directly, or by using a coordinate transformation $z = z' + \int_0^t b(\tau)\, d\tau$ in the corresponding problem of (1.1) — (1.5) where $b(t) = 0$.

Definition of Weak Solution: Let

$$H(u) = \begin{cases} \alpha_1 u & u > 0 \\ [-\alpha, 0] & u = 0 \\ \alpha_2 u & u < 0 \end{cases}$$

Assume that h is the initial temperature, ξ_0 is an arbitrary bounded and measurable graph in $H(h)$. Then, a pair (u, ξ) is called a weak solution of (1.1) — (1.5), if

$$\xi \subset H(u) \tag{1.6}$$

$$u \in L^2(0, T; H^1(G)) \tag{1.7}$$

$$\text{the trace of } u \text{ on } \partial G \times (0, T) \text{ is } g \tag{1.8}$$

and

$$\int_{\Omega_T} \left[\xi \frac{\partial \varphi}{\partial t} + b(t)\xi \frac{\partial \varphi}{\partial z} - \nabla u \cdot \nabla \varphi\right] + \int_{G(0)} \xi_0 \varphi\, dx dz = 0, \tag{1.9}$$

holds for any $\varphi \in H^1(\Omega_T)$ with $\varphi = 0$ on $\partial_p \Omega_T$, where $\partial_p \Omega_T$ is the adjoint parabolic