JOURNAL OF PARTIAL DIFFERENTIAL EQUATIONS
J. Part. Diff. Eq., Vol. **30**, No. 1, pp. 47-63

doi: 10.4208/jpde.v30.n1.4
March 2017

Viscoelastic Wave Equation with Logarithmic Nonlinearities in $\mathbb{R}^n$

BOUHALI Keltoum[1] and ELLAGGOUNE Fateh[2,*]

[1] *Universite les freres Mentouri, Faculte des sciences exactes, Departement de Mathematiques, Constantine 25000, Algeria.*
[2] *University 8 Mai 1945-Guelma, Department of Mathematics, Guelma, Algeria.*

Received 5 September 2016; Accepted 26 January 2017

Abstract. In any spaces dimension, we use weighted spaces to establish a general decay rate of solution of viscoelastic wave equation with logarithmic nonlinearities. Furthermore, we establish, under convenient hypotheses on g and the initial data, the existence of weak solution associated to the equation.

AMS Subject Classifications: 35L05, 35L71, 35B35

Chinese Library Classifications: O175.13, O175.27, O175.29

Key Words: Lyapunov function; viscoelastic; density, decay rate; weighted spaces.

1 Introduction

It is well known that from a class of nonlinearities, the logarithmic nonlinearity is distinguished by several interesting physical properties (nuclear physics, optics, and geophysics...). We consider the following semilinear equation with logarithmic nonlinearity

$$u''-\phi(x)\left(\Delta_x u-\int_0^t g(t-s)\Delta_x u(s)\mathrm{d}s\right)=u\ln|u|^k, \tag{1.1}$$

where $x\in\mathbb{R}^n, t>0, n\geq 2, k>1$ and the scalar function $g(s)$ (so-called relaxation kernel) is assumed to satisfy (A1). The model here considered are well known ones and refer to materials with memory as they are termed in the wide literature which is concerned about their physical, mechanical behavior and the many interesting analytical problems.

*Corresponding author. *Email addresses:* `klbouhali@yahoo.fr` (K. Bouhali), `fellaggoune@gmail.com` (F. Ellaggoune)

The physical characteristic property of such materials is that their behavior depends on time not only through the present time but also through their past history.

Eq. (1.1) is equipped by the following initial data.

$$u(0,x)=u_0(x)\in\mathcal{H}(\mathbb{R}^n),\quad u'(0,x)=u_1(x)\in L^2_\rho(\mathbb{R}^n), \tag{1.2}$$

where the weighted spaces $\mathcal{H}$ is given in Definition 2.1 and the density function $\phi(x)>0,\forall x\in\mathbb{R}^n,(\phi(x))^{-1}=\rho(x)$ satisfies

$$\rho:\mathbb{R}^n\to\mathbb{R}^*_+,\quad \rho(x)\in C^{0,\tilde{\gamma}}(\mathbb{R}^n), \tag{1.3}$$

with $\tilde{\gamma}\in(0,1)$ and $\rho\in L^s(\mathbb{R}^n)\cap L^\infty(\mathbb{R}^n)$, where $s=2n/(2n-qn+2q)$.

First, the following initial boundary value problem

$$u''-\Delta_x u+\int_0^t g(t-s)\Delta_x u(s)\mathrm{d}s+h(u')=f(u),\quad x\in\Omega,t>0, \tag{1.4}$$

has been studied widely. For example [1–6] and [7], the authors investigated global existence, decay rate and blow-up of the solutions.

Studies in $\mathbb{R}^n$, we quote essentially the results of [8–12]. In [10], authors showed that, for compactly supported initial data and for an exponentially decaying relaxation function, the decay of the energy of solution of a linear Cauchy problem (1.1), (1.2) with $\rho(x)=1$ is polynomial. The finite-speed propagation is used to compensate for the lack of Poincare's inequality. In [9], author looked into a linear Cauchy viscoelastic problem with density. His study included the exponential and polynomial rates, where he used the spaces weighted by density to compensate for the lack of Poincare's inequality. The same problem treated in [9], was considered in [11], where they consider a Cauchy problem for a viscoelastic wave equation. Under suitable conditions on the initial data and the relaxation function, they prove a polynomial decay result of solutions. Conditions used, on the relaxation function g and its derivative g' are different from the usual ones.

The problem (1.1), (1.2) without term source, for the case $\rho(x)=1$, in a bounded domain $\Omega\subset\mathbb{R}^n,(n\geq 1)$ with a smooth boundary $\partial\Omega$ and g is a positive nonincreasing function was considered in [12], where they established an explicit and general decay rate result for relaxation functions satisfying:

$$g'(t)\leq -H(g(t)),\ t\geq 0,\quad H(0)=0, \tag{1.5}$$

for a positive function $H\in C^1(\mathbb{R}^+)$ and H is linear or strictly increasing and strictly convex C^2 function on $(0,r],1>r$. This improves the conditions considered in [8] on the relaxation functions

$$g'(t)\leq -\chi(g(t)),\quad \chi(0)=\chi'(0)=0, \tag{1.6}$$

where χ is a non-negative function, strictly increasing and strictly convex on $(0,k_0]$, $k_0>0$. The goal of the present paper is to establish the existence of a weak solution to the problem (1.1)-(1.2). We obtain also, decay results.