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A Regularity Criterion in Terms of Direction of Vorticity to the 3D Micropolar-fluid Equations

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Abstract. In this paper, we study the regularity of weak solutions to the 3D Micropolar-fluid equations. We show that the weak solutions actually is strong solution if the corresponding vorticity field $j=\nabla\times u$ satisfies certain condition in the high vorticity region.

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1 Introduction

In this paper, we consider the Cauchy problem for the 3D Micropolar-fluid(MP) equations:

$$\begin{cases}\partial_t u+(u\cdot\nabla)u-\Delta u+\nabla p=\frac{1}{2}\nabla\times w,\\ \partial_t w-\Delta w-\nabla(\nabla\cdot w)+w+u\cdot\nabla w=\frac{1}{2}\nabla\times u,\\ \nabla\cdot u=0,\\ u(x,0)=u_0(x),\quad w(x,0)=w_0(x),\end{cases}\tag{1.1}$$

where $u(x,t)$, $w(x,t)$ are the unknown velocity field and micro-rotational velocity field; $p=p(x,t)$ is the unknown scalar pressure. While $u_0(x)$, $w_0(x)$ are the given initial data with $\nabla\cdot u_0=0$ in the sense of distribution.

Micropolar fluid system was firstly developed by Eringen [1,2]. It is a type of fluids which exhibits microrotational effects and microrotational inertia and can be viewed as

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a non-Newtonian fluid. It can describe many phenomena that appear in a large number of complex fluids such as the suspensions, animal blood, and liquid crystals which cannot be characterized appropriately by the Navier-Stokes system and that is important to the scientists working with the hydrodynamic-fluid problems and phenomena. The existences of weak and strong solutions for micropolar fluid equations were treated by Galdi and Rionero [3] and Yamaguchi [4] , respectively. The uniqueness of strong solutions to the micropolar flows and the magnetomicropolar flows either local for large data or global for small data is considered in [5,6] and references therein.

In the present paper, we consider the problem of the regularity of weak solutions to 3D MP equations. We observe that the equations include as a particular case the classical Navier-Stokes(NS) equations, which widely has been studied. There is a large literature on the problem of regularity of weak solutions to the 3D NS equations. It may be superuous to recall all results. To go directly to the main points of the present paper, we only review some known results which are closely related to our main result.

The study of conditions which involving the direction of voriticity and its physical-geometric interpretation, started with Constantin and Fefferman [7], who first derived some exact formulas and employed them in order to prove regularity in the whole space. They pointed out in the 3D case, NS equations exists a stretching mechanism for the vorticity magnitude which is non-linear and potentially capable of producing finite time singularities. They showed that the solution is smooth, if the direction of vorticity is sufficiently well behaved in the region of high vorticity magnitude. It is proved that if the direction field $\tilde{j}(x,t)$ of the vorticity $j(x,t)=\nabla\times u(x,t)$ satisfies that

$$|\tilde{j}(x+y,t)-\tilde{j}(x,t)|\le\rho|y|,\qquad t\in[0,T]. \tag{1.2}$$

for some constant $\rho>0$ when $|j(x,t)|>K$, $|j(x+y,t)|>K$ for some $K>0$; then the solution is smooth.

There are many improved and extended results to (1.2), we can see [9–13] etc. Especially, H Beirão da Veiga [9] proved that the following condition implies regularity:

Assumption H: For some $\beta\in[\frac{1}{2},1]$ and $g\in L^a(0,T;L^b)$, where

$$\frac{2}{a}+\frac{3}{b}=\beta-\frac{1}{2},\qquad a\in\left[\tfrac{4}{2\beta-1},\infty\right],$$

The weak solutions satisfy

$$|\tilde{j}(x+y,t)-\tilde{j}(x,t)|\le g(t,x)|y|^{\beta},\qquad t\in[0,T], \tag{1.3}$$

when $|j(x,t)|>K,|j(x+y,t)|>K$ for some $K>0$.

In [13], He and Xin gave a stronger condition than (1.3) to 3D MHD equations.It is worthy to emphasize that extend (1.3) to MHD equations is difficult because of the strong