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Transportation Inequalities for Multivalued Stochastic Evolution Equations

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Abstract. In this paper, using the Girsanov transformation argument, we establish Talagrand-type T_2 inequalities under the d_2 metric and the uniform metric d_∞ for the law of the solution of a class multivalued stochastic evolution equations.

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1 Introduction

Let (E,d) be a metric space equipped with a σ-field $\mathcal{B}$ such that the distance d is $\mathcal{B}\otimes\mathcal{B}$-measurable. Given $p\geq 1$ and two probability measure μ and ν on E, the Wasserstein distance is defined by

$$W_p^d(\mu,\nu)=\inf_{\pi\in(\mu,\nu)}\left(\int\int d(x,y)^p \mathrm{d}\pi(x,y)\right)^{1/p},$$

where $\mathcal{C}(\mu,\nu)$ denotes the totality of probability measures on $E\times E$ with the marginal μ and ν. In many practical situations, it is quite useful to find an upper bound for the metric $W^{p,d}(\mu,\nu)$, where a fully satisfactory one given by Talagrand [1] is the relative entropy of ν with respect to μ

$$\mathbf{H}(\nu|\mu)=\begin{cases}\int \ln\frac{\mathrm{d}\nu}{\mathrm{d}\mu}\mathrm{d}\nu, & \nu\ll\mu,\\ +\infty, & \text{otherwise,}\end{cases}$$

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where $\nu \ll \mu$ means that ν is absolutely continuous with respect to μ. We say that the probability measure μ on (E,d) satisfies the L^p transportation cost inequality (TCI) with a constant $C \geq 0$ if

$$W_p^d(\mu,\nu) \leq \sqrt{2C\mathbf{H}(\nu|\mu)}$$

for any probability measure ν. As usual, we write $\mu \in T_p(C)$ for this relation.

Concentration inequalities and their applications have become an integral part of modern probability theory. One of the powerful tools to show such concentration estimates for diffusions is TCI, where quadratic TCI is unique in its advantages and related to the log-Sobolev inequality, hypercontractivity, Poincaré inequality, inf-convolution, and Hamilton-Jacobi equations, for details, see, e.g., Bobkov and Gotze [2], Gozlan et al. [3] and Otto and Villani [4]. There are a lot of works on this subject, beginning by the contributions of Talagrand [1]. In [5] Pal showed that probability laws of certain multidimensional semimartingales satisfy quadratic TCI, in [6] Üstünel proved TCI for the laws of diffusion processes where the drift depends on the full history. In [7], Djellout et al. established the T_2 transportation inequality with respect to the Cameron-Martin metric by means of the Girsanov transform. Using Girsanov theorem, Bao and Yuan [8] and Bao et al. [9] established TCIs for neutral functional SDEs and Saussereau [10] established the transportation cost inequality for a class of SDEs driven by a fractional Brownian motion with Hurst parameter $H > 1/2$.

On the other hand, multivalued equations attracted the interest of many researchers recently. See Krée [11], Cépa [12], Bensoussan and Rascanu [13], Céping [14], and Zhang [15]. Inspired with these works, we are interested in multivalued stochastic evolution equations. To our best knowledge, there is no work reported on the TCIs for the solution of multivalued stochastic evolution equations. To close the gap in this paper, we are desired to establish TCIs for the solution of these equations by means of the Girsanov transform under the uniform metric and L^2 metric.

The rest of this paper is organized as follows. In Section 2, we introduce the multivalued stochastic evolution equation. In Section 3, we state and prove our main results.

2 Multivalued stochastic evolution equations

Let V be a separable and reflexive Banach space which is continuously and densely embedded in a separable Hilbert space H. Then we have an evolution triplet (V,H,V^*) satisfying

$$V \subset H = H^* \subset V^*,$$

where V^* is the dual space of V and we identify H with its own dual H^*.

Denote by $|\cdot|_V$, $|\cdot|_H$, $|\cdot|_{V^*}$ the norms in V, H and V^* respectively; by $\langle\cdot,\cdot\rangle_H$ the inner product in H, and ${}_V\langle\cdot,\cdot\rangle_{V^*}$ the dual relation between V and V^*. In particular, if $v \in V$ and $h \in H$, then

$${}_V\langle v,h\rangle_{V^*} = \langle v,h\rangle_H.$$