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Mixed Layer Problem of a Three-Dimensional Drift-Diffusion Model for Semiconductors

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Abstract. The quasineutral limit and the mixed layer problem of a three-dimensional drift-diffusion model is discussed in this paper. For the Neumann boundaries and the general initial data, the quasineutral limit is proven rigorously with the help of the weighted energy method, the matched asymptotic expansion method of singular perturbation problem and the entropy production inequality.

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1 Introduction

The drift-diffusion model is the most widely used model to describe semiconductor devices today [1]. It is a semi-classical macroscopic transport model and can be derived from the semiconductor Boltzmann equation [2]. The scaled three-dimensional bipolar drift-diffusion equations with the homogeneous Newmann boundary conditions and the general initial data read as follows:

$$\partial_t n^\lambda - \mathrm{div}(\nabla n^\lambda - n^\lambda \nabla V^\lambda) = 0, \qquad x\in\Omega,\ t>0, \tag{1.1}$$

$$\partial_t p^\lambda - \mathrm{div}(\nabla p^\lambda + p^\lambda \nabla V^\lambda) = 0, \qquad x\in\Omega,\ t>0, \tag{1.2}$$

$$\lambda^2 \nabla V^\lambda = n^\lambda - p^\lambda - D(x), \qquad x\in\Omega,\ t>0, \tag{1.3}$$

$$\nabla n^\lambda\cdot\overrightarrow{n}=0,\ \ \nabla p^\lambda\cdot\overrightarrow{n}=0,\ \ \nabla V^\lambda\cdot\overrightarrow{n}=0, \qquad x\in\partial\Omega,\ t>0, \tag{1.4}$$

$$n^\lambda(x,0)=n_0^\lambda(x),\quad p^\lambda(x,0)=p_0^\lambda(x), \qquad x\in\overline{\Omega}. \tag{1.5}$$

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where $\Omega = T^2 \times (0,1) \in R^3$, the variables $n^\lambda, p^\lambda, V^\lambda$ are the electron density, the hole density and the electric potential, respectively. The constant λ is the Debye length, the scaled physical parameter. The vector $\overrightarrow{n}$ is the unit outward normal vector along the boundary $\partial\Omega$. The doping profile $D = D(x)$ is a given function, and typically changes its sign. Here, we assume that $D(x)$ is a smooth function and the initial data $n_0^\lambda(x)$, $p_0^\lambda(x)$ are smooth functions satisfying

$$\int_\Omega (n_0^\lambda(x) - p_0^\lambda(x) - D(x))\mathrm{d}x = 0.$$

Physically, the Debye length $\lambda^2 \approx 10^{-7} \ll 1$, and in this case the density of electrons almost equals to the density of holes, the zero Debye length limit $\lambda \to 0$ can be also called quasineutral limit. Letting $\lambda = 0$, we formally get the following quasineutral drift-diffusion model:

$$\partial_t n^0 = \mathrm{div}(\nabla n^0 + n^0 \varepsilon^0), \qquad x \in \Omega,\ t > 0, \tag{1.6}$$

$$\partial_t p^0 = \mathrm{div}(\nabla p^0 - p^0 \varepsilon^0), \qquad x \in \Omega,\ t > 0, \tag{1.7}$$

$$0 = n^0 - p^0 - D(x), \qquad x \in \Omega,\ t > 0, \tag{1.8}$$

$$(\nabla n^0 + n^0 \varepsilon^0) \cdot \overrightarrow{n}|_{\partial\Omega} = 0, \quad (\nabla p^0 - p^0 \varepsilon^0) \cdot \overrightarrow{n}|_{\partial\Omega} = 0, \qquad x \in \partial\Omega,\ t > 0, \tag{1.9}$$

$$n^0(x,0) = n_0^0(x), \quad p^0(x,0) = p_0^0(x), \qquad x \in \overline{\Omega}. \tag{1.10}$$

The initial data $n_0^0(x)$, $p_0^0(x)$ are smooth functions satisfying the compatibility condition:

$$n_0^0 - p_0^0 - D(x) = 0.$$

The aim of this paper is to justify rigorously the limit that $(n^\lambda, p^\lambda, -\nabla V^\lambda) \to (n^0, p^0, \varepsilon^0)$ as $\lambda \to 0$.

The quasineutral limit problem is physically interesting (see [3]) and mathematically challenging (see [1,4]). Gasser et al. [5] studied the quasineutral limit of the time-dependent bipolar drift-diffusion-Poisson system, where no rigorous results on the quasineutral limit had been available before 2001. Jüngel and Peng [6] focused on the boundary layer problem of drift-diffusion-Poisson system in a bounded domain with mixed Dirichlet-Neumann boundary condition and initial conditions. The limit of the vanishing Debye length in a non-linear bipolar drift-diffusion model for semiconductors without a PN-junction was studied in [7]. Schmeiser [8] considered the initial value problem and quasineutral problem of the drift-diffusion model with the sign unchanging doping profile. For sign-changing and smooth doping profile with "good" boundary conditions, the quasineutral limit is strictly proved by introducing a "density" transform and Liapunov-type function firstly [9]. Based on [9], Wang [10] considered the multi-dimensional model and Wang [4,11] studied the limits in the spatial mean square norm uniformly in time and super-norm uniformly in space. Ju et al. [12] was devoted to the justification of the initial layer problem of bipolar transient drift-diffusion models in the multi-dimensional space.