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Existence of μ-Pseudo Almost Periodic Solutions to Some Classes of Nonautonomous Partial Evolution Equations

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Abstract. In this paper, we use a recent works [5], where the authors provide a new approach for pseudo almost periodic solution under the measure theory, under Acquistpace-Terreni conditions, we make extensive use of interpolation spaces and exponential dichotomy techniques to obtain the existence of μ-pseudo almost periodic solutions to some classes of nonautonomous partial evolution equations.

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Key Words: evolution family; exponential dichotomy; Acquistapace and Terreni conditions; almost periodic; pseudo almost periodic under the light measure; evolution equation; nonautonomous equation.

1 Introduction

In this work, we propose to study the existence of μ-pseudo almost periodic solutions under the measure theory to the class of abstract nonautonomous differential equations

$$\frac{d}{dt}\Big[u(t)+f(t,B(t)u(t))\Big]=A(t)u(t)+g(t,C(t)u(t)),\ t\in\mathbb{R}, \tag{1.1}$$

where $A(t)$ for $t\in\mathbb{R}$ is a family of closed linear operators on $D(A(t))$ satisfying the well-known Acquistapace-Terreni conditions, $B(t)$, $C(t)$ ($t\in\mathbb{R}$) are families of (possibly unbounded) linear operators, and $f:\mathbb{R}\times\mathbb{X}\mapsto\mathbb{X}_\beta^t$, $g:\mathbb{R}\times\mathbb{X}\mapsto\mathbb{X}$ are μ-pseudo almost periodic

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in $t\in\mathbb{R}$ uniformly in the second variable. Recall that the concept of μ-pseudo almost periodicity introduced by [5] is a natural generalization of the classical concept of weighted pseudo almost periodicity in the sense of Diagana [12, 13]. In recent paper [11], results on the existence and uniqueness of weighted pseudo almost periodic solutions for equation (1.1) are developed. Classical definition and properties of μ-pseudo almost periodic function solutions introduced in [5] are used.

The organization of this works is as follows. In section 2, we introduce the basic notations and recall the definitions and lemmas of μ-pseudo almost periodic functions introduced in [5], and we introduce the basic notations of evolution family and exponential dichotomy. Some preliminary results on intermediate spaces are also stated there. In Section 3, we study the existence and uniqueness of μ-pseudo almost periodic mild solution of (1.1).

2 Preliminaries

2.1 μ-pseudo almost periodic functions

Let $(\mathbb{X},||.||)$, $(\mathbb{Y},||.||)$ be two Banach spaces, and $BC(\mathbb{R},\mathbb{X})$ (respectively, $BC(\mathbb{R}\times\mathbb{Y},\mathbb{X})$) be the space of bounded continuous functions $f:\mathbb{R}\longrightarrow\mathbb{X}$ (respectively, $f:\mathbb{R}\times\mathbb{Y}\longrightarrow\mathbb{X}$). $BC(\mathbb{R},\mathbb{X})$ equipped with the norm $||f||=\sup_{t\in\mathbb{R}}||f(t)||$ is a Banach space. $B(\mathbb{X},\mathbb{Y})$ denotes the Banach spaces of all bounded linear operator from $\mathbb{X}$ into $\mathbb{Y}$ equipped with natural topology. If $\mathbb{Y}=\mathbb{X}$, $B(\mathbb{X},\mathbb{Y})$ is simply denoted by $B(\mathbb{X})$.

Definition 2.1. ([6,7]) *A continuous function $f:\mathbb{R}\mapsto\mathbb{X}$ is said to be almost periodic if for every $\epsilon>0$ there exists a positive number l such that every interval of length l contains a number τ such that*

$$\|f(t+\tau)-f(t)\|<\epsilon \ \ for \ \ t\in\mathbb{R}.$$

The set of all almost periodic functions from $\mathbb{R}$ to $\mathbb{X}$ will be denoted by a continuous function $f:\mathbb{R}\times\mathbb{Y}\mapsto\mathbb{X}$ is said to be almost periodic in t uniformly for $y\in\mathbb{Y}$, if for every $\epsilon>0$, and any compact subset K of $\mathbb{Y}$, there exists a positive number l such that every interval of length l contains a number τ such that

$$\|f(t+\tau,y)-f(t,y)\|<\epsilon \ \ for \ \ (t,y)\in\mathbb{R}\times K.$$

We denote the set of such functions $APU(\mathbb{R}\times\mathbb{Y};\mathbb{X})$.

Notice that $(AP(\mathbb{R};\mathbb{X}),\|.\|_\infty)$, is a Banach space with supremum norm given by

$$\|u\|_\infty=\sup_{t\in\mathbb{R}}\|u(t)\|.$$

Next, we give the new concept of the ergodic functions developed in [5], and generalizing the ergodicity given before [12, 13].