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Evolution Solutions of Perturbed Khokhlov-Zabolotskaya Equation

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Abstract. This paper is concerned with the exact solutions of Khokhlov-Zabolotskaya (KZ) equation with general perturbation. With the help of appropriate transformations and assumptions, the wave theory of Hopf equation is applied to get partial exact solutions. In addition, some examples and numerical simulations are presented to illustrate our analytical results.

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1 Introduction

The canonical equation for weakly nonlinear and weakly diffracting waves is the Khokhlov-Zabolotskaya (KZ) equation [1], which can be written in the general form

$$(u_t+\alpha u u_x)_x+\frac{1}{2}u_{yy}=0, \tag{1.1}$$

where $u=u(x,y,t)$ is typically a measure of the wave disturbance, x is a spatial variable measured in a frame moving with the wave, y is a transverse spatial variable, and t is a time-like variable. Then constant α is a quadratic nonlinearity parameter. It is well known that (1.1) provides an accurate description of the evolution of many systems including those corresponding to acoustic waves in air and water, shallow water waves, acoustic and magnetosonic waves in in nonlinear medium without dispersion or absorption. The study of numerous approximations to the KZ equation in (1.1) has a prominent history

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concerning the symbiotic interaction of mathematical model and scientific computing to gain insight in the topic.

Actually, for nonlinear optics, Alfvén waves in magnetohydrodynamics, and shear waves in an isotropic nonlinear solid, the quadratic nonlinearity coefficient α is small or vanishing altogether, (1.1) reduces to

$$u_t+\frac{1}{2}u_{yy}\approx 0.$$

However, if the initial wavefront is curved, near the focal region nonlinear effects will become noticeable. Consequently, Zabolotskaya [2] derived the explicit form of the extension of (1.1) for the case of shear waves propagating in a nonlinear solid in the undisturbed state. Due to the net effect of perturbation analysis, the uu_x term in (1.1) is replaced by the cubic term. (1.1) enjoys a new version

$$(u_t+\alpha u^2u_x)_x+\frac{1}{2}u_{yy}=0.$$

which was investigated by Kluwick-Cox [3] and Cramer-Webb [4]. Afterwards, a series of extension systems with weakly relaxing, weakly dissipative, and weakly dispersive were developed, one can refer to [5–9] and the latest result [10]. In this paper, we consider a perturbed acoustic wave equation with general nonlinear term and mixed derivative,

$$\big(u_t+f(u)(u_x+\gamma u_y)\big)_x+\theta u_{yy}+\frac{1}{2}\triangle_\perp u=0, \tag{1.2}$$

where $f(u)=1+u^n$, which is an arbitrary function of u. γ and θ are real constants, γ decides the propagation direction on xOy plane, and $\triangle_\perp$ denotes the transverse Laplacian in Cartesian coordinates. Our goal is to present a simple and direct method of finding partial exact solutions of KZ type equations with the help of the Hopf equation, which is available to classical KZ equation (1.1).

The rest of this paper is organized as follows. In Section 2, some elementary definitions of Hopf equation have been presented and the implicit solutions of generalized KZ equation are given by mathematical analysis. In Section 3, some examples and relevant numerical simulations are shown at the end of the paper.

2 Outline of the derivation

Here we first recall the solution of Hopf equation. As the special case of Burgers model [11], the general Hopf equation enjoys the following form:

$$u_t+\tilde{f}(u)u_x=0. \tag{2.1}$$