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Finite Time Blow Up of Solution for 1-D Nonlinear Wave Equation of Sixth Order with Linear Restoring Force at High Energy Level

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Abstract. This paper is concerned with the Cauchy problem for some 1-D nonlinear wave equations of sixth order with linear restoring force. By utilizing the concavity method and the technique of anti-dissipativity a finite time blow up result of certain solutions with arbitrarily positive initial energy is presented.

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Key Words: Cauchy problem; wave equation; sixth order; blow up; arbitrarily positive initial energy.

1 Introduction

In this paper, we consider the Cauchy problem for the following 1-D nonlinear wave equation of sixth order

$$u_{tt}-au_{xx}+u+u_{xxxx}+u_{xxxxtt}=f(u_x)_x, \qquad (x,t)\in\mathbb{R}\times(0,\infty), \tag{1.1}$$
$$u(x,0)=u_0(x),\ u_t(x,0)=u_1(x), \qquad x\in\mathbb{R}, \tag{1.2}$$

where $f(u)=b|u|^p$, $b>0$ and $p>1$ are constants, u_0 and u_1 are given initial data and $a>0$ is a given constant specified later.

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In the study of a weakly nonlinear analysis of elastoplastic-microstructure models for longitudinal motion of an elasto-plastic bar in [1] there arose the model equation

$$u_{tt}+\alpha u_{xxxx}=\beta(u_x^2)_x,$$

where $u(x,t)$ is the longitudinal displacement, and $\alpha>0$, $\beta\neq 0$ are any real numbers. The author in [1] proved the instability of the special solution and the instability of the ordinary strain solution. Subsequently Eq. (1.1) was discovered by Rosenau [2] when he was concerned with the problem that how to describe the dynamics of a dense lattice by a continuum method. Meanwhile one-dimensional homogeneous lattice wave propagation phenomena can also be described by Eq. (1.1). Later on Han [3] considered the initial boundary value problem for a class of nonlinear wave equations of the following form

$$u_{tt}-a_1u_{xx}+a_2u_{xxxx}+a_3u_{xxxxtt}=\phi(u_x)_x+f(u,u_x,u_{xx},u_{xxx},u_{xxxx}),$$

where $a_1,a_2,a_3>0$ are constants, $\phi(s)$ is an given nonlinear function. By the extension theorem they proved the existence and uniqueness of classical global solution. Moreover some sufficient conditions for finite time blow-up of the solution were also given. Recently, Wang [4] considered the Cauchy problem for the following 1-D nonlinear wave equations of higher order

$$u_{tt}-au_{xx}+u_{xxxx}+u_{xxxxtt}=f(u_x)_x.$$

By the contraction mapping principle, they proved the existence and the uniqueness of the local solution to Cauchy problem (1.1) and (1.2). By means of the potential well method, they discussed the existence and nonexistence of the global solutions to this problem under the case $E(0)\leq d$. Although as for the Cauchy problem (1.1) and (1.2), up to now, there have only been a few results, however there are many works about the global well-posedness of solutions for the nonlinear wave equation of high order [5–7] and most of them are about the low initial energy blowup.

Recently Kutev [8] investigated the affection of the linear restoring force on the well-posedness of solutions for some Boussinesq equations and showed the finite time blow up of solutions with arbitrarily positive initial energy. Inspired by this, in the present paper, we discuss the finite time blow-up of the solution to Cauchy problem (1.1) and (1.2) with the linear restoring force at high energy level. Utilizing the technique of [9] and the so-called concavity method, which was first introduced by Levine [10, 11], we show that if the initial data satisfy some conditions, then the corresponding weak solution with arbitrarily positive initial energy blows up in finite time.

The plan of the paper is as follows: In Section 2, we introduce some notations, functionals and preliminary lemmas for proving the main theorem. In Section 3, we show the main theorem of this paper and prove it.