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Long-Time Behavior for the Navier-Stokes-Voight Equations with Delay on a Non-smooth Domain

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Abstract. This paper concerns the long-time behavior for the 2D incompressible Navier-Stokes-Voight equations with distributed delay on a non-smooth domain. Under some assumptions on the initial datum and the delay datum, the existence of compact global attractors is obtained.

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1 Introduction

In this paper, we discuss the long-time behavior for the 2D Navier-Stokes-Voight equations with a distributed delay on a Lipschitz domain Ω

$$\begin{cases} \dfrac{\partial u}{\partial t}-\nu\Delta u-\alpha^2\Delta u_t+(u\cdot\nabla)u+\nabla p=f(x)+\displaystyle\int_{-h}^{0}G(s,u(t+s))\mathrm{d}s, & (x,t)\in\Omega_\tau,\\ \mathrm{div}u=0, & (x,t)\in\Omega_\tau,\\ u(x,t)|_{\partial\Omega}=\varphi,\ \ \varphi\cdot n=0, & (x,t)\in\partial\Omega_\tau,\\ u(x,\tau)=u_0(x), & x\in\Omega,\\ u(x,t)=\phi(x,t-\tau), & (x,t)\in\Omega_{\tau h}, \end{cases} \tag{1.1}$$

where $\Omega_\tau=\Omega\times(\tau,+\infty)$, $\partial\Omega_\tau=\partial\Omega\times(\tau,+\infty)$, $\Omega_{\tau h}=\Omega\times(\tau-h,\tau)$. The function $u=u(x,t)=(u_1(x,t),u_2(x,t))$ is the unknown velocity field of the fluid, p is the pressure, $\varphi\in L^\infty(\partial\Omega)$,

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$\int_{-h}^{0} G(s,u(t+s))\mathrm{d}s$ is the distributed delay, and $h>0$ is a constant. For any $t\in(\tau,T)$, we can define $u:(\tau-h,T)\to(L^2(\Omega))^2$, and $u^t(s)$ is a function defined on $(-h,0)$ satisfying $u^t(s)=u(t+s)$.

Based on the wellposedness of the Navier-Stokes equations, the infinite dimensional dynamical systems have attracted many mathematicians' attentions, which can be found in [1–3], and relating conclusions to the Navier-Stokes-Voight equations can be seen in [4–6]. The delay effect was investigated first for ordinary differential equations in physics, for the results on the Navier-Stokes equations with delays on bounded domain, such as the existence of global solutions and attractors, we can refer to [7–14].

The research on dynamical system for nonlinear dissipative system on the non-smooth domains (such as the Lipschitz domain) is a difficult problem, some interesting results can be seen in [15–18], and in this paper we shall use the method of [15] to derive the existence of compact global attractors.

This paper is organized as follows. In Section 2, some preliminaries are given which will be used in the sequel. The existence and uniqueness of solutions for system (1.1) are derived in Section 3. And in the last section, we derive the existence of global attractors in an appropriate topology space.

2 Preliminaries

2.1 Functional spaces

Denote $E:=\{u|u\in(C_0^\infty(\Omega))^2, \mathrm{div}u=0\}$, H denotes $\overline{E}$ in $(L^2(\Omega))^2$ topology, $|\cdot|$ and $(\cdot,\cdot)$ represent the norm and inner product in H respectively, where

$$|u|=\Big(\int_\Omega |u|^2\mathrm{d}x\Big)^{1/2},\ (u,v)=\sum_{j=1}^{2}\int_\Omega u_j(x)v_j(x)\mathrm{d}x, \qquad \forall\, u,v\in H, \tag{2.1}$$

and V denotes $\overline{E}$ in $(H^1(\Omega))^2$ topology, and $\|\cdot\|$ and $((\cdot,\cdot))$ denote the norm and inner product in V respectively, where

$$\|u\|=\Big(\int_\Omega |\nabla u|^2\mathrm{d}x\Big)^{1/2},\ ((u,v))=\sum_{i,j=1}^{2}\int_\Omega \frac{\partial u_j}{\partial x_i}\frac{\partial v_j}{\partial x_i}\mathrm{d}x, \qquad \forall\, u,v\in V. \tag{2.2}$$

Also, V' is the dual space of V with norm $\|\cdot\|_*$. Write

$$C_H=C^0([-h,0];H),\ C_V=C^0([-h,0];V)$$

with norms $\|u\|_{C_H}=\sup_{\theta\in[-h,0]}|u(t+\theta)|$ and $\|u\|_{C_V}=\sup_{\theta\in[-h,0]}\|u(t+\theta)\|$ respectively. Similarly, $L_H^2=L^2(-h,0;H)$.