JOURNAL OF PARTIAL DIFFERENTIAL EQUATIONS
J. Part. Diff. Eq., Vol. **31**, No. 4, pp. 374-384

doi: 10.4208/jpde.v31.n4.7
December 2018

Spherically Symmetric Solutions for a Coupled Compressible Navier-Stokes/Allen-Cahn System

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Received 9 October 2018; Accepted 2 November 2018

Abstract. In this paper, we consider the initial boundary value problem of a coupled compressible Navier-Stokes/Allen-Cahn system which is described the motion of a mixture of two viscous compressible fluids in 3D. Our aim is to show the existence and uniqueness of local classical solution under the assumption of spherically symmetric condition for initial data ρ_0 without vacuum state.

AMS Subject Classifications: 35Q30, 35Q35, 35B65, 76D05, 76F65

Chinese Library Classifications: O175.29

Key Words: Navier-Stokes equations; Allen-Cahn equation; local solutions; spherically symmetry.

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1 Introduction

As we know, the interface between fluids is usually assumed to be separated by a sharp interface in classical models. But when the sharp interface is replaced by a narrow transition layer in some cases, we can introduce a phase field variable χ and define a mixing energy by χ and its spatial gradient. Therefore, this model is a strong coupling systems between the Navier-Stokes equations and the Allen-Cahn equations by the Cauchy stress tensor.

In this paper, we shall consider the model proposed by Blesgen [1], which had been investigated in [2] as follows.

$$\partial_t\rho+\mathrm{div}_x(\rho\mathbf{u})=0, \tag{1.1}$$

$$\partial_t(\rho\mathbf{u})+\mathrm{div}_x(\rho\mathbf{u}\otimes\mathbf{u})=\mathrm{div}_x(\mathbb{T}), \tag{1.2}$$

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$$\partial_t(\rho\chi)+\mathrm{div}_x(\rho\chi\mathbf{u})=-\mu(\rho,\chi,\Delta\chi), \tag{1.3}$$

$$\rho\mu=-\delta\Delta\chi+\rho\frac{\partial f(\rho,\chi)}{\partial\chi}, \tag{1.4}$$

where $\rho,\mathbf{u}$ and χ denote the density function, the velocity filed, and the phase variable, respectively. The Cauchy stress tensor satisfies

$$\mathbb{T}=\mathbb{S}-\delta\left(\nabla_x\chi\otimes\nabla_x\chi-\frac{|\nabla_x\chi|^2}{2}\mathbb{I}\right)-p(\rho,\chi)\mathbb{I},$$

$$\mathbb{S}=\nu(\chi)\left(\nabla_x\mathbf{u}+\nabla_x^k\mathbf{u}-\frac{2}{3}\mathrm{div}_x\mathbf{u}\mathbb{I}\right)+\eta(\chi)\mathrm{div}_x\mathbf{u}\mathbb{I},$$

where $p=\rho^2\frac{\partial f(\rho,\chi)}{\partial\rho}$ is the thermodynamic pressure, and $f(\rho,\chi)$ is the potential energy density.

If we take the density ρ be a positive constant, then the system above reduces to an incompressible one. The diffuse interface models for two-phase flows of incompressible fluids have been extensively studied. For the coupled Navier-Stokes/Allen-Cahn system with $\frac{\partial f}{\partial\chi}=\frac{1}{\delta}(\chi^3-\chi)$, where δ is a positive constant and $\sqrt{\delta}$ represents the thickness of the interface, Zhao et al. [3] investigated the vanishing viscosity limit and they proved that the solutions of the Navier-Stokes/Allen-Cahn system converge to that of the Euler/Allen-Cahn system in a proper small time interval. Babak et al. [4] provided a unified and comparative description of the most prominent phase field based two-phase flow models and presented the numerical results of the application of Galerkin-based isogeometric analysis to incompressible Navier-Stokes/Cahn-Hilliard equations in velocity-pressure-phase field-chemical potential formulation. Xu et al. [5] established axisymmetric solutions to coupled incompressible Navier-Stokes/Allen-Cahn equations. Zhang [6] established a regularity criterion for the 3D incompressible density-dependent Navier-Stokes-Allen-Cahn system. Moreover, for the Navier-Stokes/Cahn-Hilliard system and numerical simulations, such as jet pinching-off and drop formation, we refer the readers to [7–11].

As far as we know, there are less theoretical available results for the compressible models. For the Navier-Stokes/Allen-Cahn system, Chen et al. [12] investigated the global existence and uniqueness of strong and classical solutions of 1D initial boundary value problem. Feireisl et al. [2] proved the existence of weak solutions in 3D, where the density ρ is a measurable function. A different compressible Navier-Stokes/Allen-Cahn system arising from the biological material change in the process differentiation had been studied in [13]. Thanks to the ideas of the investigation on the liquid crystals, Ding et al. [14] have obtained the global solutions for a coupled compressible Navier-Stokes/Allen-Cahn system in 1D. For the Navier-Stokes/Cahn-Hilliard system, we can refer the references [15–17] and therein.

For the compressible Navier-Stokes equations, the pressure p is usually chosen as $p=A\rho^\gamma$ with $A>0$ and $\gamma>1$, and for Allen-Cahn equation, double-well structural potential