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Principal Eigenvalue for Cooperative (p,q)-biharmonic Systems

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Abstract. In this article, we are interested in the simplicity and the existence of the first strictly principal eigenvalue or semitrivial principal eigenvalue of the (p,q)-biharmonic systems with Navier boundary conditions.

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1 Introduction

Let $\Omega \subset \mathbb{R}^N$ (with $N \geq 1$) be a bounded domain with smooth boundary $\partial\Omega$ and α, β, p, q be constants such that $\alpha \geq 0$, $\beta \geq 0$, $p > 1$, $q > 1$ and $\dfrac{\alpha+1}{p} + \dfrac{\beta+1}{q} = 1$.

Our aim is to study the following eigenvalue problem

$$(Q): \begin{cases} \Delta_p^2 u - \lambda m_1(x)|u|^{p-2}u = m(x)|v|^{\beta+1}|u|^{\alpha-1}u & \text{in } \Omega, \\ \Delta_q^2 v - \lambda m_2(x)|v|^{q-2}v = m(x)|u|^{\alpha+1}|v|^{\beta-1}v & \text{in } \Omega, \\ u = \Delta u = v = \Delta v = 0 & \text{on } \partial\Omega, \end{cases}$$

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where $\Delta_p^2 u = \Delta(|\Delta u|^{p-2}\Delta u)$ is the p-biharmonic operator and λ is a real parameter. The coefficients m_1, m_2, $m \in L^\infty(\Omega)$ are assumed to be nonnegatives in Ω.

In [1], Talbi and Tsouli have investigated the scalar version of problem (Q) with $m \equiv 0$, which reads

$$(P_{a,p,\rho}): \begin{cases} \Delta(\rho|\Delta u|^{p-2}\Delta u) = \lambda a(x)|u|^{p-2}u & \text{in } \Omega, \\ u = \Delta u = 0 & \text{on } \partial\Omega, \end{cases}$$

where $\rho \in C(\overline{\Omega})$ such that $\rho > 0$ and $a \in L^\infty(\Omega)$. They proved that $(P_{a,p,\rho})$ possesses at least one non-decreasing sequence of eigenvalues and studied $(P_{a,p,\rho})$ in the particular one dimensional case. The authors, in the same reference gave the first eigenvalue $\lambda_{1,p,\rho}(a)$ and showed that if $a \geq 0$ a.e. in Ω, then $\lambda_{1,p,\rho}(a)$ is simple (i.e. the associated eigenfunctions are a constant multiple of one another) and principal i.e. the associated eigenfunction, denoted by $\varphi_{p,\rho,a}$ is positive or negative on Ω with

$$\lambda_{1,p,\rho}(a) = \inf_{u \in \mathcal{A}} \int_\Omega \rho |\Delta u|^p \mathrm{d}x, \tag{1.1}$$

where

$$\mathcal{A} = \left\{ u \in W^{2,p}(\Omega) \cap W_0^{1,p}(\Omega) : \int_\Omega a|u|^p \mathrm{d}x = 1 \right\}. \tag{1.2}$$

The problem $(P_{a,p,\rho})$ was considered by P. Drábek and M. Ôtani for $\rho \equiv 1$ and $a \equiv 1$ [2]. By using a transformation of the problem to a known Poisson problem, they showed that $(P_{a,p,\rho})$ has a principal positive eigenvalue which is simple and isolated. In the case $N = 1$ they gave a description of all eigenvalues and associated eigenfunctions.

El Khalil et al. [3] also considered problem $(P_{a,p,\rho})$ for $\rho \equiv 1$, $a \equiv 1$ with Dirichlet boundary conditions and showed that the spectrum contains at least one non-decreasing sequence of positive eigenvalues.

Benedikt [4] gave the spectrum of the p-biharmonic operator with Dirichlet and Neumann boundary conditions in the case $N = 1$, $\rho \equiv 1$ and $a \equiv 1$.

It is important to note that (u,λ) is solution of problem $(P_{m_1,p,1})$ if and only if $[(u,0);\lambda]$ is solution of (Q). This kind of solution is called "semitrivial solution" of (Q). Furthermore if $[(u,0);\lambda]$ is solution of (Q) with u of one sign on Ω, then λ is called "semitrivial principal eigenvalue" of (Q). Consequently, there are two forms of semitrivial solutions for problem (Q): one of the type $[(u,0);\lambda]$ with $u \not\equiv 0$ and (u,λ) solution of the problem $(P_{m_1,p,1})$ and the second of the type $[(0,v);\lambda]$ with $v \not\equiv 0$ and (v,λ) solution of the problem $(P_{m_2,q,1})$. In particular $\lambda_{1,p,1}(m_1)$ and $\lambda_{1,q,1}(m_2)$ are semitrivial principal eigenvalues of (Q).

This paper is organized as follows. We construct the eigencurve associated to problem (Q) in Section 2. Section 3 is devoted to the study of strictly principal eigenvalue of (Q).