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# Well-Posedness for the 2D Non-Autonomous Incompressible Fluid Flow in Lipschitz-like Domain

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**Abstract.** This paper is concerned with the global well-posedness and regularity of weak solutions for the 2D non-autonomous incompressible Navier-Stokes equation with a inhomogeneous boundary condition in Lipschitz-like domain. Using the estimate for governing steady state equation and Hardy's inequality, the existence and regularity of global unique weak solution can be proved. Moreover, these results also hold for 2D Navier-Stokes equation with Rayleigh's friction and Navier-Stokes-Voigt flow, but invalid for three dimension.

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**Key Words**: Non-autonomous Navier-Stokes equation; Lipshitz-like domain; background flow function.

## 1 Introduction

The incompressible Navier-Stokes equation is a well-known hydrodynamical model which plays a important role in understanding continuous medium mechanics. Our objective in this paper is to study the global well-posedness and its regularity for a 2D incompressible non-autonomous fluid flow with non-homogeneous boundary condition in a

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Lipschitz-like domain:

$$\begin{cases} \dfrac{\partial u}{\partial t} - \nu\Delta u + (u\cdot\nabla)u + \nabla p = f(t,x), & x\in\Omega, t\geq\tau, \\ \mathrm{div} u = 0, & x\in\Omega, t\geq\tau, \\ u=\varphi, \quad \varphi\cdot n=0, & x\in\partial\Omega, t\geq\tau, \\ u(\tau,x)=u_\tau(x), & x\in\Omega, \end{cases} \tag{1.1}$$

wxhere $\Omega\subset\mathbb{R}^2$ is a bounded set which is said to be a Lipschitz-like domain if its boundary $\partial\Omega$ can be covered by finitely many balls $B_i=B(Q_i,r_0)$ centered at the point $Q_i\in\partial\Omega$ such that for each ball $B_i$, there exists a rectangular coordinate system and a Lipschitz function $\Psi:\mathbb{R}^{d-1}\to\mathbb{R}$ with

$$B(Q_i,3r_0)\cap\Omega = \Big\{(x_1,x_2,\cdots,x_d)\,\Big|\, x_d>\Psi_i(x_1,x_2,\cdots,x_{d-1})\Big\}\cap\Omega,$$

$\tau\in\mathbb{R}$ is an initial time. The variables $u$ represents the fluid velocity field, $p$ denotes the pressure, and $\nu$ is the kinematic viscosity. In addition, $n$ represents the exterior unit normal vector to $\partial\Omega$, $\varphi=\varphi(x)$ is a prescribed tangential boundary velocity, and $f(t,x)$ is a time-dependent forcing term.

The 2D incompressible Navier-Stokes equations with homogeneous Dirichlet or periodic boundary in smooth domain, we can refer to literature [3–7, 11]. And the 2D autonomous system with non-homogenous boundary on smooth domain can be founded in [8, 9]. For this problem extended to non-smooth, by an appropriate background flow, [1] investigated the well-posedness in less regular space and its dynamics. In this paper, we want to investigate the non-autonomous case. Firstly, we introduce the background function $\psi$, which is the solution to following problem that shares the same boundary condition $\varphi$ as (1.1):

$$\begin{cases} \mathrm{div}\ \psi=0, & \text{in } \Omega, \\ \psi=\varphi, \quad \varphi\cdot n=0 & \text{on } \partial\Omega. \end{cases} \tag{1.2}$$

and $\psi$ is a solution for the Stokes problem

$$\begin{cases} -\triangle\hat{u}+\nabla q=0, & \text{in } \Omega, \\ \mathrm{div}\ \hat{u}=0, & \text{in } \Omega, \\ \hat{u}=\varphi, \ a.e. \text{ on } \partial\Omega \text{ in the sense of nontangential convergence.} \end{cases} \tag{1.3}$$

The idea comes from [8] and [9], then [1] extended to non-smooth domains of 2D Navier-Stokes equation by critically invoking estimates of the Stokes problem. Then, writing