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SOME PROBLEMS OF NONLINEAR SCHRÖDINGER EQUATIONS WITH THE EFFECT OF DISSIPATION

Guo Boling
(Institute of Applied Physics and Computational Mathematics)
Wang Lireng
(Department of Mathematics, East China Normal Univ.)

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Abstract We first consider the initial value problem of nonlinear Schrödinger equation with the effect of dissipation, and prove the existence of global generalized solution and smooth solution as some conditions respectively. Secondly, we disscuss the asymptotic behavior of solution of mixed problem in bounded domain for above equation. Thirdly, we find the "blow up" phenomenon of the solution of mixed problem for equation

$$iu_t = \Delta u + \beta f(|u|^2)u - i\frac{\gamma(t)}{2}u, \qquad x \in \Omega \subset \mathbf{R}^3, t > 0$$

i. e. there exists $T_0 > 0$ such that $\lim_{t\to T_0} \| \nabla u \|^2_{L_2(\Omega)} = \infty$. The main means are a prior estimates on fractional degree Sobolev space, related properties of operator's semigroup and some integral identities

Key Words effect of dissipation; global generalized solution; global smooth solution; asymptotic behavior; blow up; Sobolev inequality; strong differential function; optimal constant.

Classification 35Q20.

1. Introduction

The nonlinear Schrödinger equation occurs in many of physical problems; thin beem current of nonlinear optics; Langmuir waves in plasmas; one dimensional self modulation of monochromatic wave; stationary two dimensional self-focusing of aplane wave; Ginzbug-Landau equation of superconducting-electron pair moves on the electromagnetic case and so on.

The global solution, asymptotic behavior and "blow up" phenomenon for the solution of its initial value problems and initial-boundary value problems have been studied by many authors, for example, see[1-5]and [6].

One of the authors of this paper discussed the global solution of initial value problem for the system of nonlinear Schrödinger equation with the magnetic field effect on R^2, and found the "blow up" phenomenon of spherically symmetric solution for the above system on R^3 under some conditions [7]. In this paper we first consider the initial value problem of nonlinear Schrödinger equation with effect of dissipation

$$\begin{cases} iu_t = \Delta u + \beta f(|u|^2)u - i\frac{\gamma}{2}g(|u|^2)u, & x \in R^3, t > 0 \quad (1.1) \\ u|_{t=0} = u_0(x), & x \in R^3 \quad (1.2) \end{cases}$$

and discuss the existence of the global solution for problem (1.1)(1.2). Secondly, we consider the mixed problem

$$\begin{cases} iu_t = \Delta u + \beta f(|u|^2)u - i\frac{\gamma}{2}g(|u|^2), & x \in \Omega \subset R^3, t > 0 \quad (1.1)' \\ u|_{t=0} = u_0(x), & x \in \Omega; u|_{\partial\Omega} = 0, \quad t \geqslant 0 \quad (1.3) \end{cases}$$

and the asymptotic estimates of its solution is obtained. Thirdly, we consider the mixed problem

$$\begin{cases} iu_t = \Delta u + \beta f(|u|^2)u - i\frac{\gamma(t)}{2}u, & x \in \Omega \subset R^3, t > 0 \quad (1.4) \\ u|_{t=0} = u_0(x), x \in \Omega; \quad u|_{\partial\Omega} = 0, \quad t \geqslant 0 \quad (1.5) \end{cases}$$

and find that the "blow up" phenomenon still occurs in the solution for nonlinear Schrödinger equation with the effect of dissipation.

In this paper, the proof is established by means of integral estimates on fractional degree Sobolev space, related properties of operator's semigroup and some integral identities.

The notations used in this paper are: Ω shows the bounded domain in R^3; $\partial\Omega$ shows the boundary of Ω; $H^m(R^3)$ shows the Sobolev space, for function $u \in H^m(\Omega)$, its norm is show by $\| u \|_{H^m(\Omega)} = (\sum_{|\alpha| \leqslant m} \| D^\alpha u \|^2_{L_2(\Omega)})^{1/2}$, and the norm of function $u \in H^m(R^3)$ has the similar definition; $H_0^m(\Omega)$ shows the closure in H^m of $C_0^\infty(\Omega)$; $\| u \|_{L_\infty(R^3)} = \operatorname{ess\,sup}_{x \in R^3} |u(x)|$; for other notations, see [8].

2. The Global Solution of the Initial Problem (1.1)(1.2)

We first consider the initial problem (1.1)(1.2) of nonlinear Schrödinger equation with the effect of dissipation $-i\frac{\gamma}{2}g(|u|^2)u$, here β and γ are real numbers.

Lemma 1 *If (1) $\gamma \geqslant 0, g(s) \geqslant 0, s \in [0, \infty)$; (2) $u_0(x) \in L_2(R^3)$; then for the solution $u(x,t)$ of problem (1.1)(1.2), we have*