CORRECTION TO "HIGHER-ORDER NONLINEAR SYSTEM OF EQUATIONS OF CHANGING TYPE"

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The proof of Lemma 2 is incorrect, because the boundary values of u_{x^M} are not given. To correct this error we replace Lemmas 2, 3, 4 by the following lemma:

Lemma 2′ *Under the conditions in* (3) *there is the uniform estimate*

$$\begin{aligned} &\| u_{x^M t} \|^2_{L_2(Q_t)} + \| u_{x^{2M}} \|^2_{L_2(Q_t)} + \| u_{tt} \|^2_{L_2(Q_t)} \\ &\leqslant C\{1 + |\varphi|^2_{H^{2M}(0,1)} + \| f \|^2_{H^1(Q_T)}\}, \quad \forall\, t \in [0,T] \end{aligned} \tag{12}$$

Proof Firstly, making the integration $[(Lu)_t, -b(t)u_{tt}](t), \forall\, t>t_0$, we have

$$\begin{aligned} &-\frac{1}{2}(b(t)K(t)u_{tt}, u_{tt})(t) + \frac{1}{2}(b(0)K(0)u_{tt}(0,x), u_{tt}(0,x)) \\ &+\left[\left(-\frac{3}{2}b(t)K'(t) + \frac{1}{2}b'(t)K(t)\right)u_{tt}, u_{tt}\right](t) - [b(t)K''(t)u_t, u_{tt}](t) \\ &+\frac{1}{2}(b(t)Au_{x^M t}, u_{x^M t})(t) - \frac{1}{2}(b(0)Au_{x^M t}(0,x), u_{x^M t}(0,x)) \\ &-\frac{1}{2}[b'(t)Au_{x^M t}, u_{x^M t}](t) + [\mathscr{H}(u)u_t, b(t)u_{tt}](t) \\ &= [f_t, -b(t)u_{tt}](t), \forall\, t > t_0 \end{aligned} \tag{13}$$

where $\mathscr{H}(u) = \left(\dfrac{\partial^2 F(u)}{\partial u_i \partial u_j}\right)$ is the Hessian matrix of $F(u)$. The term on the right-hand side in (13) is estimated by

$$|[f_t, -b(t)u_{tt}]| \leqslant \frac{k_0}{6} \| b(t)^{1/2} u_{tt} \|^2_{L_2(Q_t)} + C \| f_t \|^2_{L_2(Q_t)} \tag{14}$$

The 4-th term on the left-hand side in (13) is estimated by

$$|[b(t)K''(t)u_t, u_{tt}]| \leqslant \frac{k_0}{6} \| b(t)^{1/2} u_{tt} \|^2_{L_2(Q_t)} + C \| b^{1/2} u_t \|^2_{L_2(Q_t)} \tag{15}$$

The 3rd term on the left-hand side in (13) is estimated by

$$\begin{aligned} &\left[\left(-\frac{3}{2}b(t)K'(t) + \frac{1}{2}b'(t)K(t)\right)u_{tt}, u_{tt}\right] \\ &\geqslant \int_{t_0}^{t}\int_0^1 b(t)\left(\frac{3}{2}k_0 I - \lambda K(t)\right)u_{tt} \cdot u_{tt}\, dx dt \end{aligned}$$

$$+\int_{0}^{t_0}\int_{0}^{1} b(t)\left(\frac{3}{2}k_0 I-\varepsilon K(t)\right)u_{tt}\cdot u_{tt}dxdt$$

$$\geqslant k_0[b(t)u_{tt},u_{tt}](t),\quad \forall\, t>t_0 \tag{16}$$

In consideration of the assumption (3)(iii) we obtain

$$|[\mathscr{H}(u)u_t,b(t)u_{tt}]|\leqslant C_5[|u|^\rho|u_t|,b(t)|u_{tt}|]+C_6[|u_t|,b(t)|u_{tt}|]$$

$$\leqslant C_5\||u|^\rho\|_{L_p(Q_t)}\|b(t)^{1/2}u_t\|_{L_q(Q_t)}\|b(t)^{1/2}u_{tt}\|_{L_2(Q_t)}$$

$$+\frac{k_0}{6}\|b(t)^{1/2}u_{tt}\|^2_{L_2(Q_t)}+C_6(k_0)\|b(t)^{1/2}u_t\|^2_{L_2(Q_t)}$$

where $\frac{1}{p}+\frac{1}{q}=\frac{1}{2}$, $p>2$ (any real number), and $q=\frac{2p}{p-2}$. From (11) we have

$$|[\mathscr{H}(u)u_t,b(t)u_{tt}]|\leqslant\frac{k_0}{3}\|b(t)^{1/2}u_{tt}\|^2_{L_2(Q_t)}+C_6'(k_0)\|b(t)^{1/2}u_t\|^2_{L_2(Q_t)}$$

$$+C_5'(k_0)\|b(t)^{1/2}u_t\|^2_{L_q(Q_t)}$$

By Nirenberg's inequality

$$\|b(t)^{1/2}u_t\|_{L_q(Q_t)}\leqslant C\|b(t)^{1/2}u_t\|^{1-a}_{L_2(Q_t)}\|b(t)^{1/2}u_t\|^{a}_{H^1(Q_t)}$$

$$\leqslant C'\|b(t)^{1/2}u_t\|^{a}_{H^1(Q_t)} \tag{17}$$

where $a=1-2/q=2/p<1$. Hence

$$|[\mathscr{H}(u)u_t,b(t)u_{tt}]|\leqslant\frac{k_0}{3}\|b(t)^{1/2}u_{tt}\|^2_{L_2(Q_t)}$$

$$+\eta(\|b(t)^{1/2}u_{tt}\|^2_{L_2(Q_t)}+\|b(t)^{1/2}u_{xt}\|^2_{L_2(Q_t)})+C(k_0,\eta) \tag{18}$$

The positivity of the 1st, 2nd, 5th and 7th terms on the left-hand side in (13) hold by the assumptions (3) and (7). Then, in consideration of (14)-(18), from (13) it follows the estimation

$$\frac{k_0}{3}\|b(t)^{1/2}u_{tt}\|^2_{L_2(Q_t)}+\frac{a_0}{2}|b(t)^{1/2}u_{x^\kappa t}(t,\cdot)|^2_{L_2(0,1)}+\frac{\varepsilon a_0}{2}\|b(t)^{1/2}u_{x^\kappa t}\|^2_{L_2(Q_t)}$$

$$\leqslant\frac{b(0)a_1}{2}|u_{x^\kappa t}(0,x)|^2_{L_2(0,1)}+\eta(\|b(t)^{1/2}u_{tt}\|^2_{L_2(Q_t)}+\|b(t)^{1/2}u_{xt}\|^2_{L_2(Q_t)})$$

$$+C\{\|f\|^2_{H^1(Q_T)}+|\varphi|^2_{H^\kappa(0,1)}+1\},\quad\forall\, t>t_0 \tag{19}$$

Since