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REFLECTION OF PROGRESSING WAVES FOR QUASILINEAR HYPERBOLIC SYSTEMS

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Abstract It is proved that when a progressing wave described by conormal distribution for a quasilinear hyperbolic 2×2 system hits a solid wall transversally, the reflected wave remains conormal. In contrast to the semilinear case, such conormal distributions had to be defined inductively to take into account of the fact that the relevant characteristic surfaces are not necessarily smooth. The argument involves a suitable coordinate change to reduce the problem to a simple form and an iterative induction on the tangential regularity of the solution as well as that of the characteristic surfaces at which the wave fronts are situated.

Key Words quasilinear hyperbolic systems; reflection of waves; singularities; conormal distributions.

Classifications 35L60; 35B65.

1. Introduction

This paper deals with the problem of transversal reflection of progressing waves for quasilinear hyperbolic 2×2 systems. For semilinear prxoblems, much work has been done by J. Berning & M. Reed[5], M.Oberguggenberger[12] in one space dimensional case, and by M. Beals & G. Metivier[3,4], Chen Shuxing[7], Gu Ben[9], Wang Yaguang[13] in higher dimensional case.

The main difficulty for quasilinear problems lies in the fact that the characteristic surfaces of quasilinear equations depend on the solution, so the smoothness of the characteristic surfaces is not guaranteed if the solution has singularities. Consequently, the associated conormal distributions which are used to describe the progressing waves cannot be defined in the same way as in semilinear case. S. Alinhac[1,2] has solved the problem of propagation and interaction of waves for quasilinear equations using the machinery of para-composition and para-differential calculus developed by him. In this paper, we adopt a direct definition of conormal distributions motivated by Alinhac's and prove with simple techniques a theorem on the reflection of singularities for quasilinear hyperbolic 2×2 systems.

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2. Distribution Spaces

Let M be a bounded region in R^m. The set of all vector fields on M with $H^s(M)$ coefficients is denoted by $H^s(M, TM)$, of which the element is also viewed as first order differential operator with $H^s(M)$ coefficients. In this paper, $C^\infty(M)$ is understood as the space of restrictions of smooth functions on R^m. Suppose $\mathcal{V}$ is a finitely generated $C^\infty(M)$ -submodule of $H^s(M, TM)$.

Definition 2.1 *Assume $s > \frac{m}{2}$. Define $\mathcal{H}^{r,0}(M,\mathcal{V}) = H^r(M)$, $0 \le r \le s$. For integer $k \ge 1$, if $\mathcal{H}^{r,k-1}(M,\mathcal{V})$ has been defined for $0 \le r \le s$ and if the coefficients of any vector field in $\mathcal{V}$ belong to $\mathcal{H}^{s,k-1}(M,\mathcal{V})$, then define inductively*

$$\mathcal{H}^{r,k}(M,\mathcal{V}) = \{v \in \mathcal{H}^{r,k-1}(M,\mathcal{V}) : Vv \in \mathcal{H}^{r,k-1}(M,\mathcal{V}) \text{ for any } V \in \mathcal{V}\}$$
$$0 \le r \le s$$

In general, $\mathcal{H}^{r,k}(M,\mathcal{V})$ can only be defined to some integer k_0. The situation that $\mathcal{H}^{s,k}(M,\mathcal{V})$ has the definition will mean that the coefficients of vector fields in $\mathcal{V}$ belong to $\mathcal{H}^{s,k-1}(M,\mathcal{V})$. If $\mathcal{V} \subset C^\infty(M, TM)$, the space $\mathcal{H}^{r,k}(M,\mathcal{V})$ has the definition for any integer $k \ge 0$ in view of the following lemma.

Lemma 2.2 *If $s > \frac{m}{2}$ and $\mathcal{H}^{r,k}(M,\mathcal{V})$ has definition for $0 \le r \le s$, then*

(i) $\mathcal{H}^{s,k}(M,\mathcal{V}) \cdot \mathcal{H}^{r,k}(M,\mathcal{V}) \subset \mathcal{H}^{r,k}(M,\mathcal{V})$, $0 \le r \le s$;

(ii) $C^\infty(M) \subset \mathcal{H}^{s,k}(M,\mathcal{V})$.

Proof The first conclusion is the consequence of the fact that $H^s(M) \cdot H^r(M) \subset H^r(M)$. To prove the second one, note that $C^\infty \subset \mathcal{H}^{s,0}$. Now assume $C^\infty \subset \mathcal{H}^{s,j}$ ($j \le k-1$), so for $\phi \in C^\infty$,

$$\phi, \partial_{x_1}\phi, \ldots, \partial_{x_m}\phi \in \mathcal{H}^{s,j}$$

If $V = \sum_{i=1}^{m} a_i \partial_{x_i} \in \mathcal{V}$, then the condition of the lemma guarantees that $a_i \in \mathcal{H}^{s,k-1}$, and (i) implies

$$V\phi = \sum_{i=1}^{m} a_i \partial_{x_i} \phi \in \mathcal{H}^{s,j}$$