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TRAVELLING WAVE FRONT SOLUTIONS FOR REACTION-DIFFUSION SYSTEMS[1]

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Abstract In this paper by using upper-lower solution method, under appropriate assumptions on f and g the existence of travelling wave front solutions for the following reaction-diffusion system is proved:

$$\begin{cases} u_t - u_{xx} = f(u,v) \\ v_t - v_{xx} = g(u,v) \end{cases}$$

As an application, the necessary and sufficient condition of the existence of monotone solutions for the boundary value problem

$$\begin{cases} u'' + cu' + u(1-u-rv) = 0 \\ v'' + cv' - buv = 0 \\ u(-\infty) = v(+\infty) = 0 \\ u(+\infty) = v(-\infty) = 1 \end{cases}$$

where $0 < r < 1$, $0 < b < \dfrac{1-r}{r}$ are known constants and c is unknown constant to be obtained.

Key words Reaction-diffusion system; travelling wave front solutions; upper-lower solution method; B–Z reaction.

Classification 35K55.

1. Introduction

In recent years there has been considerable interest in travelling wave front solutions $(u(x-ct), v(x-ct))$ of the following reaction-diffusion system

$$\begin{cases} u_t - u_{xx} = f(u,v) \\ v_t - v_{xx} = g(u,v) \end{cases} \tag{1}$$

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Different kinds of methods for proving the existence of travelling wave front solutions have been developed, for examples:

1. Perturbation method (Asymptotic analysis method). For details, see [3-1, 2].

2. Shooting method by solving boundary value problem for ordinary differential equations, see [2], [6].

3. Travelling wave front solutions as limits of solutions of boundary value problems on finite domains. This method has been used by [1-1, 2] and [9]. On the other hand, [9] is different from [1], in the latter upper-lower solution method is used; in the former topological degree theory is used.

4. Conley index method, see [4].

If solutions of (1) of the form $u(x,t) = u(x-ct)$, $v(x,t) = v(x-ct)$ in which $u(\xi), v(\xi) (\xi = x - ct)$ are both bounded, monotone, and nonconstant, they are called travelling wave front solutions (TWFS) of (1). For TWFS (u, v) the limits

$$u(\pm\infty) = u_\pm, \qquad v(\pm\infty) = v_\pm$$

exist and $u_+ \neq u_-, v_+ \neq v_-$.

In this paper, first we will discuss the general properties of travelling wave solutions of (1), then we will develop the upper-lower solution method to prove the existence of TWFS for general system (1), at last, as an application of our method, we will discuss the existence of TWFS for the system

$$\begin{cases} u_t = u_{xx} + u(1-u-rv) \\ v_t = v_{xx} - buv \end{cases}$$

Throughout this paper we always suppose that f and g are at least continuous.

2. Basic Properties of Travelling Wave Solutions

By a simple calculation we know that $u = u(x-ct)$, $v = v(x-ct)$ is a travelling wave solution of (1) if and only if $(u(\xi), v(\xi))$ is a solution for the following ordinary differential equations

$$\begin{cases} -u'' - cu' = f(u,v) \\ -v'' - cv' = g(u,v) \end{cases} \qquad \xi \in R^1 \tag{2}$$

where $' = \frac{d}{d\xi}$, $\xi = x - ct$.

Lemma 2.1 *Let $(u(\xi), v(\xi))$ be a solution of (2) with*

$$u(\pm\infty) = u_\pm, \qquad v(\pm\infty) = v_\pm \tag{3}$$

then

$$\lim_{\xi\to\pm\infty} (u'(\xi), v'(\xi)) = (0,0)$$