J. Partial Differential Equations
Vol. 4, No. 3 (1991), 73–78

EXISTENCE OF CLASSICAL SOLUTION OF A QUASILINEAR STEFAN-SIGNORINI PROBLEM

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(Received Sept. 20, 1988; revised Sept. 25, 1989)

Abstract In this paper, we consider an one-phase quasilinear Stefan-Signorini problem. In [1] X. Liu has proved the existence and the uniqueness of this problem by the traditional penalty method (see [3] and [4]). Here we use the Mixed Scheme in [2] which is more suitable to the Stefan-Signorini problem and obtain more natural existence results.

Key words Quasilinear Stefan-Signorini problem; mixed scheme; priori estimates.

Classification . 35K.

1. The Problem and the Results

To find a pair of functions $\{u(x,t), s(t)\}$, $u(x,t)$: $\bar{Q} \to R^1$, $s(t) : [0,T] \to [0,1]$, satisfying

$$a(x,t,u)u_{xx} + b(x,t,u)(u_x)^2 + c(x,t,u)u_x + d(x,t,u)u - u_t = f(x,t)$$

$$s(t) < x < 1, \qquad 0 < t < T \tag{1.1}$$

$$u(x,t) = \phi(x), \qquad 0 \le x \le 1 \tag{1.2}$$

$$u_x(1,t) = 0, \qquad 0 < t < T \tag{1.3}$$

$$u(s(t),t) \ge 0, \qquad 0 < t < T \tag{1.4}$$

$$s'(t) = g(t) - u_x(s(t),t), \qquad 0 < t < T \tag{1.5}$$

$$s'(t)u(s(t),t) = 0, \qquad 0 < t < T \tag{1.6}$$

$$s(0) = 0 \tag{1.7}$$

where $Q \equiv \{(x,t)|0 < t < T, s(t) < x < 1\}$.

The solution $\{u(x,t),\, s(t)\}$ is classical in the sense that $s(t)\in C^1([0,T])$, $u\in C(\bar{Q})$, $u_x\in C(\bar{Q})$, $u_{xx}\in C(Q)$ and $u_t\in C(Q)$.

Assume that $g(t)\in C^1([0,T_0])$, $0<T_0\le\infty$, $\phi(x)\in C^2([0,T)]$, the coefficients of (1.1) have continuous derivatives of up to the third order in all their variables, and $\phi(x)\ge 0, 0\le x\le 1$, $g(0)-\phi'(0)\ge 0$, $\phi(0)[g(0)-\phi'(0)]=0$, $d(x,t,\xi)\le\gamma$, $f(x,t)\le 0$, $a(x,t,\xi)\ge\lambda>0$, where γ,λ and T_0 are given constants.

Moreover, we assume that either of the two following conditions holds:

[A] $\quad g(t)\ge 0, t\in[0,T]$, or

[B] $\quad b(x,t,\xi)\le 0,\quad a(x,t,\xi)\le\gamma,\quad |c(x,t,\xi)|\le\gamma$

Theorem *Under the above assumptions, for any $\varepsilon\in(0,1)$, there is a $T(0<T\le T_0)$ depending only on ε, T_0 and $g(t)$, such that the problem (1.1)–(1.7) has at least one classical solution. In addition, $0\le s(t)\le 1-\varepsilon$, $0\le t\le T$, $s(t)\in C^{1+\frac{1}{2}}([0,T])$, $u_{xx}\in L^\infty(Q)$ and $u_t\in L^\infty(Q)$.*

2. The Mixed Scheme

Auxiliary Problem (I) *Given a function $u^{n-1}(x)$ and a constant s_{n-1}, $0\le s_{n-1}<1$, $u^{n-1}(x)\in C^1([s_{n-1},1])$, $u^{n-1}\ge 0$, find a function $u^n(x)$ and a number s_n, such that $u^n(x)\in C^3([s_n,1])$, $s_{n-1}\le s_n<1$, and satisfy*

$$a(x,t_n,u^n)u^n_{xx}+b(x,t_n,u^n)(u^n_x)^2+c(x,t_n,u^n)u^n_x+d(x,t_n,u^n)u^n = f(x,t_n)+\frac{1}{h}(u^n-u^{n-1}),\qquad s_n<x<1 \tag{2.1}$$

$$u^n(s_n)\ge 0 \tag{2.2}$$

$$u^n(s_n)\Delta_n=0 \tag{2.3}$$

$$\Delta_n\ge 0 \tag{2.4}$$

$$u^n_x(1)=0 \tag{2.5}$$

$$s_n=s_{n-1}+h\Delta_{n-1} \tag{2.6}$$

where $\Delta_n=g(t_n)-u^n_x$, $t_n=nh$, $n=0,1,\cdots,N$, $N=\left(\frac{T}{h}\right)$, $h\le h_0, h_0$ and T are constants to be determined.

Auxiliary Problem (II) *Under the assumptions of the auxiliary problem (I), to find $u^n(x)$ and s_n such that $s_{n-1}\le s_n<1$, $u^n(x)\in C^3([s_n,1])$, and satisfy (2.1)–(2.5) and*

$$s_n=s_{n-1}+h\Delta_n \tag{2.7}$$