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# THE GLOBAL SOLUTIONS OF A NONLINEAR SYSTEM OF EQUATIONS OF CHANGING TYPE*

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**Abstract** In this paper, the global existence of regular solutions to the initial-boundary value problem for a higher-order multidimensional system of equations of changing type with a strong nonlinear term is studied.

**Key Words** Equations of changing type; multidimensional system; global existence; fixed point principle.

**Classification** 35M05

In domain $Q_T \equiv \{(t,x)|0 \le t \le T, x = (x_1,\cdots,x_n) \in \Omega \subset R^n\}$ we consider the nonlinear system of equations of changing type

$$Lu \equiv \Big(K(t)u_t\Big)_t + (-1)^{M-1} \sum_{|\alpha|,|\beta|=M} D_x^\alpha (A_{\alpha\beta}(x) D_x^\beta u) - \operatorname{grad} F(u) = f(t,x) \quad (1)$$

and initial-boundary value problem

$$\begin{cases} D^\nu u = 0, \quad 0 \le |\nu| \le M-1 \quad \text{on } \partial\Omega \times [0,T] \\ u(0,x) = \varphi(x), \quad x \in \Omega \subset R^n \end{cases} \quad (2)$$

where $M \ge 1$ is an integer, $u$, $f$ and $\varphi$ are $N$-dimensional vectors: $u = (u_1,\cdots,u_N)$, $f = (f_1,\cdots,f_N)$, $\varphi = (\varphi_1,\cdots,\varphi_N)$; $K(t)$ is an $N \times N$ diagonal matrix: $K(t)$ =diag $\{k_1(t),\cdots,k_N(t)\}$; $A_{\alpha\beta}$ are $N \times N$ matrices; $F$ is a nonlinear function of vector $u$; $\Omega$ is a bounded domain with smooth boundary, $T$ is any finite real number.

Assume that the coefficients and functions in (1) and (2) satisfy the following conditions:

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\begin{cases}
\text{(i)} \quad k_i(t) > 0 \text{ when } t = 0,\ k_i(t) \geq 0 \text{ when } t \in (0, t_0) \\
\qquad k_i(t) = 0 \text{ when } t = t_0, k_i(t) < 0 \text{ when } t \in (t_0, T] \\
\qquad k_i(t) \in C^2[0,T],\ k_i'(t) \leq -k_0 < 0, \forall t \in [0, t_0], \quad i = 1, \cdots, N \\
\text{(ii)} \quad A_{\alpha\beta}(x) \text{ are symmetric positively definite matrices :} \\
\qquad \sigma_1 |D_x^M u|^2_{L_2(\Omega)} \geq \Big( \sum_{|\alpha|,|\beta|=M} A_{\alpha\beta}(x) D_x^\alpha u, D_x^\beta u \Big) \geq \sigma_0 |D_x^M u|^2_{L_2(\Omega)}, \quad \sigma_0, \sigma_1 > 0 \\
\text{(iii)} \quad F \in C^2, F(\varphi(x)) \in L_1(\Omega) \text{ and } F \text{ satisfies :} \\
\qquad F(u) \geq -C_1 \\
\qquad \Big| \dfrac{\partial F(u)}{\partial u_i} \Big| \leq C_2 |u|^{\rho+1} + C_3, \ i = 1, \cdots, N \\
\qquad \Big| \dfrac{\partial^2 F(u)}{\partial u_i \partial u_j} \Big| \leq C_4 |u|^\rho + C_5, \ i, j = 1, \cdots, N \\
\qquad \text{where } C_k > 0, 1 \leq k \leq 5, \text{and } \rho \text{ is a non-negative real number : } \rho < \dfrac{2}{n-1} \\
\text{(iv)} \quad \varphi_i \in H^{2M}(\Omega) \cap L_{2(\rho+1)}(\Omega), \quad f_i \in H^1(Q_T), \quad i = 1, \cdots, N
\end{cases}
\tag{3}
$$

It is evident that, in the case $M = 1$, the system (1) is a nonlinear system of equations of mixed type, which is elliptic in the domain $Q_{t_0}$, and is hyperbolic in the domain $Q_T \setminus \overline{Q}_{t_0}$, $t = t_0$ is its degenerate plane. In the case $M > 1$, system (1) is of hypoelliptic type in $Q_{t_0}$, and is of ultrahyperbolic type in $Q_T \setminus \overline{Q}_{t_0}$, hence (1) is a nonlinear system of equations of changing type.

In practical problems there appear higher-order equations of changing type[1]. There are only a few papers concerning this type of equations ([2–6]), but there is not any paper concerning the system of equations of changing type.

In the case $n = 1$ in [7] we have proved the global existence of regular solutions to the problem (1) (2) without any restriction on $\rho$.

In the case $n > 1$, the global existence of regular solutions to the problem (1) (2) is proved in [8] only for the case: $M = 1$, $2 \leq n \leq 3$, $\rho \leq \frac{2}{n}$.

In this paper we solve this problem for any $n$ and any $M$ under the restriction $\rho < \frac{2}{n-1}$.

Assume that on the degenerate plane $t = t_0$ the following normal connected condi-