J. Partial Differential Equations
Vol. 5, No. 1 (1992), 17-22

THE EXISTENCE OF TRAVELLING WAVE FRONT SOLUTIONS FOR REACTION-DIFFUSION SYSTEM*

Li Zhengyuan
(Institute of Maths., Peking University, Beijing 100871)
(Received Sept. 10, 1990)

Abstract In this paper by using upper-lower solution method the critical wave speed of wave front for a simplified mathematical model of Belousov-Zhabotinskii chemical reaction

$$u_t - u_{xx} = u(1 - u - rv)$$

$$v_t - v_{xx} = -buv$$

is obtained, where $0 < r < 1$, $b > 0$ are known.
Key Words Reaction-diffusion system; travelling wave front solutions; upper-lower solution method; B-Z reaction.
Classification 35K35

1. Introduction

In this paper we discuss the existence of travelling wave front solution of the following reaction-diffusion system

$$\begin{cases} u_t - u_{xx} = f(u, v) \\ v_t - v_{xx} = g(u, v) \end{cases} \tag{1}$$

A method for finding travelling wave front solution as limits of solutions of boundary value problems on finite domains has been developed by [1]-[4]. In the paper [4] by using the method and upper-lower solutions, a general principle for the existence of travelling wave front solution for the system (1) has been established. As an application of the general principle, necessary and sufficient condition for the existence of the monotone solution for the boundary value problem

$$\begin{cases} u'' + cu' + u(1 - r - u + rv) = 0 \\ v'' + cv' + bu(1 - v) = 0 \\ u(-\infty) = v(-\infty) = 0 \\ u(+\infty) = v(+\infty) = 1 \end{cases} \tag{2}$$

* The project partially supported by the National Natural Science Foundation of China

was obtained: for $0 < r < 1$, $0 < b < (1-r)/r$, it was proved that (2) has a monotone solution if and only if $c \le -2\sqrt{1-r}$. In this paper we extend the above result for the existence of travelling wave front solutions to a principle which is much more convenient for practical use and in case of system (2) it does not require the limitation on b:

$$b < (1-r)/r$$

This also provides a similar result for the existence of monotone solution for (2).

2. Theorem of Existence

We first state our assumptions.

(H. 1): Let $l \ge 1$ and $a > 0$ be large enough.

(i) There exist a pair of upper and lower solutions $(\bar{u}_a, \bar{v}_a)$, $(\underline{u}_a, \underline{v}_a)$ for the following system on $[-a, a]$

$$\begin{cases} u'' + cu' + f(u,v) = 0 \\ v'' + cv' + g(u,v) = 0 \end{cases}$$

where $c \in \boldsymbol{R}$ is fixed and the functions f, g are as specified below. Moreover

$$0 < \underline{u}_a \le \bar{u}_a \le l, \quad 0 \le \underline{v}_a < \bar{v}_a \le l$$

(ii) $\bar{u}_a'(\xi) \ge 0$, $\underline{u}_a'(\xi) \ge 0$, $\bar{v}_a'(\xi) \ge 0$ for any $\xi \in [-a, a]$.

(iii) For any $\varepsilon > 0$ there exists $A > 0$ such that for any $\xi \in [-a, -A]$

$$\underline{v}_a(\xi) < \varepsilon, \quad \bar{v}_a(\xi) < \varepsilon$$

when $a > A$.

(H. 2):

(i) On $[0,l] \times [0,l]$, the only solutions of the equations

$$f(u,v) = 0, \quad g(u,v) = 0$$

are $(0,\alpha)(\alpha \in [0,l])$ and $(1,1)$.

(ii) $f \in C^1([0,l] \times [0,l])$, $f(u,v) = uf_1(u,v)$, for $(u,v) \in (0,l) \times (0,l)$,

$$\frac{\partial f_1}{\partial u} < 0, \quad \frac{\partial f_1}{\partial v} > 0, \quad f_1(0,0) > 0$$

(iii) $g \in C^1([0,l] \times [0,l])$. There exists $l_0 \in (0,l]$ such that

$$g(u,v) > 0, \quad (u,v) \in (0,l) \times (0,l_0)$$

$$g(u,v) < 0, \quad (u,v) \in (0,l) \times (l_0,l), \quad \text{when} \quad l_0 < l$$