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THE NONLINEAR BOUNDARY VALUE PROBLEMS OF THREE ELEMENTS FOR THE FIRST ORDER QUASILINEAR ELLIPTIC SYSTEMS

Hu Guoen
(The Information Engineering College of the People's Liberation Army of China, Zhengzhou 450002)

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Abstract In this paper, we discuss the nonlinear boundary value problems of three elements with two shifts for the first order quasilinear elliptic systems and the related solvability by using the continuity method.

Key Words The first order quasilinear elliptic systems; nonlinear boundary value problems; continuity method.
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1. Introduction

A great number of results have been got about the boundary value problems for the first order elliptic systems[1-4].

In this paper we discuss the nonlinear boundary value problems of three elements with two shifts for the first order quasilinear elliptic systems.

Assume that Γ is a simple smooth closed curve in the complex plane E, denote by G^+ the simple connected region surrounded, denote $G^- = F \setminus G^+$, $F(z,w)$ is a complex function of complex variable z and the complex function $w(z)$, the function $g(t, w_1(t), w_2(t))$ is defined on $\Gamma \times E \times E$. And $\alpha(t)$, $\beta(t)$ are positive and opposite shifts respectively, satisfying

(a) $\alpha(\alpha(t)) \equiv \beta(\beta(t)) \equiv t, \quad \alpha(\beta(t)) \equiv \beta(\alpha(t)) \quad$ for $t \in \Gamma$;

(b) $\alpha'(t)$ and $\beta'(t)$ are Hölder continuous on Γ.

$G_1(t)$ and $G_2(t)$ are Hölder continuous functions and different from zero on Γ. Find a piecewise regular solution $w(z)$ of the first order quasilinear elliptic systems

$$w_{\bar{z}} = F(z,w) \qquad \text{for } z \in E \setminus \Gamma$$

such that it satisfies the boundary conditions

$$w^+(t) = G_1(t)w^-(\alpha(t)) + G_2(t)w^+(\beta(t)) + g(t, w^+(t), w^-(t)) \quad \text{for } t \in \Gamma$$

$$|w(z)| = O(|z|^m) \text{ for } z \to +\infty \text{ and the integer } m$$

For the related notations see [1], we assume

(c) For each fixed point z in E, $F(z,w)$ has second order continuous partial derivatives with respect to w and $\overline{w}$, which are uniformly bounded in the norm $L_{p,2}[\cdot, E]$. Denote by C this bound, $F(z,0) \in L_{p,2}(z)$, $p > 2$.

(d) For arbitrary $w_1(t), w_2(t) \in H_\nu(\Gamma)$, $g(t, w_1(t), w_2(t)) \in H_\nu(\Gamma)$, $\nu = 1 - 2/p$; and there exists a nonnegative constant M such that for any $w_1^{(1)}(t)$, $w_1^{(2)}(t)$, $w_2^{(1)}(t)$, $w_2^{(2)}(t) \in H_\nu(\Gamma)$

$$\begin{aligned} &C_\nu[g(t, w_1^{(1)}(t),\ w_2^{(1)}(t)) - g(t, w_1^{(2)}(t),\ w_2^{(2)}(t)),\ \Gamma] \\ &\qquad \le M\{C_\nu[w_1^{(1)}(t) - w_1^{(2)}(t),\ \Gamma] + C_\nu[w_2^{(1)}(t) - w_2^{(2)}(t),\ \Gamma]\} \end{aligned} \tag{1}$$

2. Linear Boundary Value Problems of Three Elements

Theorem 1 *Under the conditions that a more respective inequality is added, the piecewise regular solutions in the complex plane of the boundary value problems of generalized analytic functions*

$$\begin{cases} w_{\bar z} + Aw + B\overline{w} = 0 & \text{for } z \in E \setminus \Gamma \\ w^+(t) = w^-(\alpha(t)) + G(t)w^+(\beta(t)) & \text{for } t \in \Gamma \\ w^-(\infty) = 0 \end{cases} \tag{2}$$

$$\begin{cases} w_{\bar z} + Aw + B\overline{w} = 0 & \text{for } z \in E \setminus \Gamma \\ w^-(t) = w^+(\alpha(t)) + G(t)w^-(\beta(t)) & \text{for } t \in \Gamma \\ w^-(\infty) = 0 \end{cases} \tag{3}$$

are all unique zero solutions, where $A(z)$, $B(z) \in L_{p,2}(E)$, $p > 2$. $G(t)$ is a Hölder continuous function on Γ.

Proof Assume that $w(z)$ is a solution of the boundary value problem (2). By using the expression $w(z) = \Phi(z)e^{\omega(z)}$[1] we can obtain that $\Phi(z)$ satisfies

$$\begin{cases} \Phi^+(t) = e^{\omega^-(\alpha(t)) - \omega^+(t)}\Phi^+(\alpha(t)) + G(t)e^{\omega^+(\beta(t)) - \omega^+(t)}\Phi^+(\beta(t)) \text{ for } t \in \Gamma \\ \Phi^-(\infty) = 0 \end{cases} \tag{4}$$

Taking $H_1(t) = e^{\omega^-(\alpha(t)) - \omega^+(t)}$, $H_2(t) = e^{\omega^+(\beta(t)) - \omega^+(t)}G(t)$, we have $\mathrm{ind}_\Gamma H_1(t) = 0$ and $H_2^*(\tau)H_2^*(\beta_1(\tau)) = G^*(\tau)G^*(\beta_1(\tau))$ for $\tau \in L$. For the variable τ, the curve L, the shifts $\beta_1(\tau)$, $H_2^*(\tau)$, $G^*(\tau)$, see [5].

Let $X_1(z) = \exp\Big\{\dfrac{1}{2\pi i}\displaystyle\int_L \ln[H_1^*(\tau)]\frac{d\tau}{\tau - z}\Big\}$. From the properties of $\omega(z)$[1], it follows that there exists a positive constant M_ν depending only on the curve Γ, the shifts $\alpha(t)$, $\beta(t)$ and $L_{p,2}(|A| + |B|)$ such that $|X_1^-(\beta_1(\tau))/X_1^-(\tau)| < M_\nu$ for $\tau \in L$. So if we