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A SEMIGROUP APPROACH TO NONLINEAR EQUATION OF AGE-DEPENDENT POPULATION DYNAMICS

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Abstract This paper deals with nonlinear model, of age-dependent population dynamics, with nonautonomous time dependence. Using a nonlinear semigroup approach, the authors prove the existence of the problem under general assumptions.

Key Words Nonlinear population equation; nonautonomous equation; nonlinear evolution operators.

Classification 35L60; 35Q99.

1. Introduction

In this paper we will use a nonlinear semigroup approach to deal with the nonlinear boundary problem of age-dependent population equation:

$$\begin{cases} \dfrac{\partial l}{\partial t}+\dfrac{\partial l}{\partial a}=G(t,l), & \text{in } (O,T)\times(0,\infty) \quad (1.1)\\ l(t,0)=F(t,l), & \text{for } t\in(0,T) \quad (1.2)\\ l(0,a)=\phi(a), & \text{for } a\in(0,\infty) \quad (1.3)\end{cases}$$

where $l(t,a)=(l_1(t,a),\cdots,l_n(t,a))^T$: $[0,T]\times[0,\infty)\to \boldsymbol{R}^n$, $G(t,l)=(G_1(t,l),\cdots,G_n(t,l))^T$: $[0,T]\times L^1(0,\infty;\ \boldsymbol{R}^n)\to L^1(0,\infty;\ \boldsymbol{R}^n)$, $F(t,l)=(F_1(t,l),\cdots,F_n(t,l))^T$: $[0,T]\times L^1(0,\infty;\ \boldsymbol{R}^n)\to \boldsymbol{R}^n$ and $\phi=(\phi_1,\cdots,\phi_n)^T$.

In the autonomous case where F and G are independent of t, G. F. Webb [6] investigated problem (1.1)–(1.3) in detail by the method of integral equations. After getting the existence of the solution, he proved that the solution operator of the problem generates a nonlinear semigroup and the relevant exponential formula holds. For nonautonomous case, some authors have studied the problems with special forms of F and G (cf., e.g., [4], [5]).

The design of our work is that under more general assumptions on F and G we treat problem (1.1)–(1.3) with nonautonomous time dependence along the way inverse to that used in [6], i.e., we first prove that the family of operators $A(t)$ defined by

$$A(t)\phi = \dot{\phi} - G(t,\phi), \quad t \geq 0$$

satisfies the conditions of the theorem given by M. G. Crandall and A.Pazy [1], therefore generates a family of nonlinear evolution operators, and the exponential formula holds. Then we prove that the "weak solution" given by the exponential formula is exactly the solution of the system of the integral equations corresponding to problem (1.1)–(1.3).

In Section 2 we give the assumptions on F, G, and state the main results of the paper. In Section 3 we prove that $A(t)$ generates a family of nonlinear evolution operators. In Section 4, the main part of this paper, we prove that the function defined by the exponential formula satisfies the corresponding system of integral equations, provided F and G are Lipschitz continuous. Finally, Section 5 is devoted to show that the result is true also for F, G being only locally Lipschitz continuous.

2. Main Results

For $x = (x, \cdots, x)^T \in \boldsymbol{R}^n$, let $|x| = \sum_{i=1}^{n} |x_i|$. Let $L^1 = L^1(0,\infty;\boldsymbol{R}^n)$ be the Banach space of all the Lebesgue integrable functions from $(0,\infty)$ to $\boldsymbol{R}^n$ with norm $\|f\|_{L^1} = \int_0^\infty f(a)da$ for $f \in L^1$, and

$$L^1_+ = \{f | f = (f_1, \cdots, f_n)^T \in L^1, \quad f_i \geq 0, \ 1 \leq i \leq n\}$$

We now list the set of hypotheses on F and G.

(AF.1) There exists a function $C_F(t,r)$ defined on $[0,T]\times[0,\infty)$, which is continuous in t for fixed r and increasing in r for fixed t, such that

$$|F(t,\phi) - F(t,\psi)| \leq C_F(t,r)\|\phi - \psi\|_{L^1}$$

for any $\phi, \psi \in L^1$, $\|\phi\|_{L^1}, \|\psi\|_{L^1} \leq r$.

(AF.2) There exists a continuous function $h_F(t)$ of bounded variation on $[0,T]$, and an increasing function $L_F(r)$ on $[0,\infty)$ such that

$$|F(t_1,\phi) - F(t_2,\phi)| \leq |h_F(t_1) - h_F(t_2)| L_F(\|\phi\|_{L^1})$$

for any $\phi \in L^1$, $t_1, t_2 \in [0,T]$.

(AF.3) $F(t,\phi) \in \boldsymbol{R}^n_+$, for any $\phi \in L^1_+$, $t \in [0,T]$.

(AG.1) There exists a function $C_G(t,r)$ on $[0,T]\times[0,\infty)$, which is continuous in t for fixed r and increasing in r for fixed t, such that

$$\|G(t,\phi) - G(t,\psi)\|_{L^1} \leq C_G(t,r)\|\phi - \psi\|_{L^1}$$