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BIFURCATION FOR SOME SYSTEMS OF COOPERATIVE AND PREDATOR-PREY TYPE*

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Abstract In this paper we provide a full classification of the non-negative solutions types for a class of two-dimensional bilinear cooperative and predator-prey systems in terms of two bifurcation parameters.

Key Words Positive solutions; least eigenvalue; cooperative system; predator-prey system
Classification 35K55, 35P30.

1. Introduction

The stationary distributions of two mutually interacting populations can be described by the system of the form

$$\begin{cases} -\Delta u = u(a_{11}u + a_{12}v + b_1) & \text{in } \Omega \\ -\Delta v = v(a_{21}u + a_{22}v + b_2) & \text{in } \Omega \\ u = v = 0 & \text{on } \partial\Omega \end{cases} \tag{1}$$

where $\Omega \subset \boldsymbol{R}^n$ is a bounded domain with smooth boundary $\partial\Omega, a_{ij}$ and b_i are constants. An important problem is that of the existence of non-negative solutions, that is, the solutions have both components non-negative. Many authors have discussed this problem such as [1], [2]. The non-negative solutions of (1) can be of three different types: the trivial solution $(0,0)$, possible semitrivial solutions $(u^+, 0), (0, v^+)$ with u^+ and v^+ positive and possible nontrivial solutions (u^+, v^+) with both components positive. In this paper, a_{ij} represent given constants and we let b_i vary; our main aim is to determine under what case the non-negative solution $(u(x), v(x))$ is trivial and

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under what case (1) has semitrivial non-negative solutions or nontrivial non-negative solutions.

We shall provide a full classification of the non-negative solution types according to the value of the parameters b_1, b_2. This will be done in the following tow cases:

$$\begin{cases} -\Delta u = u(-a_{11}u + a_{12}v + b_1) & \text{in} \quad \Omega \\ -\Delta v = v(a_{21}u - a_{22}v + b_2) & \text{in} \quad \Omega \\ u = v = 0 & \text{on} \quad \partial\Omega \end{cases} \tag{2}$$

with

$$\Delta = \begin{vmatrix} -a_{11} & a_{12} \\ a_{21} & -a_{22} \end{vmatrix} > 0, \quad a_{ij} > 0 \quad (i, j = 1, 2)$$

and

$$\begin{cases} -\Delta u = u(-a_{11}u - a_{12}v + b_1) & \text{in} \quad \Omega \\ -\Delta v = v(a_{21}u - a_{22}v + b_2) & \text{in} \quad \Omega \\ u = v = 0 & \text{on} \quad \partial\Omega \end{cases} \tag{3}$$

where $a_{ij} > 0 (i, j = 1, 2)$.

Equations (2) describe the equilibrium states of a cooperative system, while (3) those of a predator-prey system (v being the predators, u the preys). As relevant parameters we choose b_1, b_2, namely the amount of available resources for the populations (notice that b_i/a_{ii}, for $i = 1, 2$ represent the carrying capacity of either population when the other one is absent).

Let a be a continuous function on the closure $\bar{\Omega}$ of Ω. We denote by $\lambda_0(a(x))$ the least eigenvalue of the problem

$$\begin{cases} -\Delta u + a(x)u = \lambda u & \text{in} \quad \Omega \\ u = 0 & \text{on} \quad \partial\Omega \end{cases}$$

Let ϕ_0 with $\max\{\phi_0(x) | x \in \bar{\Omega}\} = 1$ be the eigenfunction associated with $\lambda_0(a(x))$. (In the special case $a(x) \equiv 0$, let $\lambda_0 = \lambda_0(0)$ for simplicity). Denote by $\hat{u}(x)$ the maximal non-negative solution of the first equation in (2) or (3) with boundary condition $u|_{\partial\Omega} = 0$ when $v(x) \equiv 0$. We denote $\hat{v}(x)$ analogously by considering the second equation. It is known that $\hat{u}(x) \equiv 0 (\hat{v}(x) \equiv 0)$ if $b_1 \leq \lambda_0 (b_2 \leq \lambda_0)$, while $\hat{u}(x) > 0 (\hat{v}(x) > 0)$ in Ω if $b_1 > \lambda_0 (b_2 > \lambda_0)$.

Our main results are: