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OPTIMAL CONTROL OF A CLASS OF PHASE CHANGE PROCESSES WITH TERMINAL STATE OBSERVATION*

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Dedicated to the 70th birthday of Professor Zhou Yulin

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Abstract A class of optimal control problems for nonlinear evolutionary processes governed by two-phase Stefan problems is analyzed. The processes with terminal state observation are considered in the case of one space dimension. Approximate optimal solutions (controls, as well as the corresponding states and adjoint states), referring to the problems with time-averaged state observation are shown to converge to the appropriate solutions for the original problem.

Key Words free boundary, optimal control, terminal observation, approximate solutions

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1. Introduction

A typical feature of many optimal control problems with terminal state observation for evolutionary systems (equally nonlinear and linear), in particular of the parabolic type, consists in their numerical sensitivity to variations of the related data (cf. [4]). Not only any perturbation of the terminal state subject to observation, but also variations of the terminal time instant itself contribute to large changes of the optimal controls, regardless boundary or distributed. The terminal observation reflects a primary interest in the final product of the process, at any cost of the state trajectory leading to that product.

An idea of improving the numerical performance is based on constructing an auxiliary family of control problems with distributed observation. That observation extends

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the original component of the objective functional by its averaging in time over some terminal interval. Such an approach seems quite natural, moreover it can be proved to produce convergent algorithms. The appropriate results on the convergence of the optimal controls and the optimal pair of the primary and adjoint states are established.

We shall consider a quite general class of systems governed by nonlinear parabolic problems. To become more specific, we focus on the problems that arise from the modelling of diffusive processes connected with phase change phenomena. We assume the appropriate models in their phase-field setting (cf. [1], in particular).

We first outline the scope. The state y of a controlled system $S^T(y;f)$ with control variable f is to be determined so as to minimize a cost functional $\mathcal{J}^T(y;f)$ over a certain set of admissible controls. The functional is assumed to include a term explicitly dependent on the terminal state $y(T)$ at a given finite time instant T. Simultaneously we shall introduce an auxiliary family of relaxed control problems which, instead of the terminal observation term, include approximating it distributed terms defined over some time intervals $[T-\varepsilon, T]$. The appropriate relaxed cost functionals will have the form $\mathcal{J}^\varepsilon(y;f), \varepsilon > 0$, with the state observation extended over the whole time interval $[T-\varepsilon, T]$.

For reference purposes, let us now introduce a model for which we shall specify all results throughout this paper. The model has arisen from the phase-field approach to modelling the dynamics of phase change processes (cf. [1, 9]) and it exhibits qualitative features most characteristic for nonlinear parabolic systems of spatial pattern formation. The development of the desired spatial structures is of primary applicative significance there and gives rise to a non-academic control problem with terminal observation.

In this context, the state y of the system comprehends two components, $y = \{u, \varphi\}$. Throughout the paper, $\Omega \subset \mathbf{R}^n$ will be a given bounded open domain, with $Q = \Omega \times (0,T]$—the corresponding cylindrical domain.

Problem $S^T(f)$ *Given $T < \infty$ and f, determine the state of the system $S^T(y;f)$, defined as a pair $\{u,\varphi\}$ that satisfies the following parabolic initial-boundary value problem:*

$$\frac{\partial u}{\partial t} + l\frac{\partial \varphi}{\partial t} = \Delta u + f \quad in\ Q \tag{1.1}$$

$$\frac{\partial \varphi}{\partial t} = \Delta\varphi + \lambda(\varphi) + u \quad in\ Q \tag{1.2}$$

$$\frac{\partial u}{\partial \nu} = \frac{\partial \varphi}{\partial \nu} = 0 \quad on\ \partial\Omega \times (0,T] \tag{1.3}$$

$$u(x,0) = u_0(x) \quad in\ \Omega \tag{1.4}$$

$$\varphi(x,0) = \varphi_0(x) \quad in\ \Omega \tag{1.5}$$

where $l \geq 0$ is a given constant, and the functions $u_0, \varphi_0 \in W_2^\infty(\Omega)$ and smooth $\lambda(\varphi)$