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ON INVERSION OF PERMITIVITY, PERMEABILITY AND CONDUCTIVITY*

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Abstract In this paper, the inversion of coefficients permitivity ε, permeability μ and conductivity σ in Maxwell system is discussed. Under some smoothness conditions, we have proved the existence, uniqueness and extension theorems of the local solution to the inverse problem by means of an equivalent integral system. Stability, well-posed property of the solution and the property of the solution at the end point of the largest existence interval are also researched.

Key Words Inverse problem; Maxwell system; largest existence interval; well-posed property.

Classification 35R30.

1. Transformation and decomposition of the basic model

For the simplest layered earth model, there are many methods to determine elastic parameters and single electromagnetic parameter (for example wave speed, wave number etc.) [1–5]. It is usually based on the assumptions that conductivity $\sigma = 0$ and magnetic permeability μ =const. to determine single electromagnetic parameter. In this paper, we are going to determine the electromagnetic parameters μ, σ and permitivity ε simultaneously under some smoothness conditions.

1.1 Basic model

Let $E = (E_1, E_2, E_3)^T, H = (H_1, H_2, H_3)^T$ be the electric and magnetic field strengths, (x_1, x_2, x_3, t) point-time coordinate system. The layered earth model and its surface are denoted by the semi-space $x_3 > 0$ and the plane $x_3 = 0$ respectively. ε, μ and σ are the functions of only one variable x_3. If we suppose that the density of the external electric current is zero inside earth, Maxwell system which describes the processes of electromagnetic oscillation propagation and the basic model which we shall deal with can be written as follows:

$$\Big(A\frac{\partial}{\partial t} + \sum_{j=1}^{3} B_j \frac{\partial}{\partial x_j} + B_4\Big)U = 0, \quad x_3 > 0,\ t > 0,\ -\infty < x_1,\ x_2 < \infty \tag{1.1}$$

$$U|_{t<0} = 0 \tag{1.2}$$

$$H_1|_{x_3=0} = 0, \quad H_2|_{x_3=0} = \delta(x_1 - x_1^0,\ x_2 - x_2^0, t) \tag{1.3}$$

$$E_1|_{x_3=0} = f(x_1, x_2, t) \tag{1.4}$$

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Here $A = \begin{bmatrix} \varepsilon I_3 & 0 \\ 0 & \mu I_3 \end{bmatrix}$, $B_4 = \begin{bmatrix} \sigma I_3 & 0 \\ 0 & 0 \end{bmatrix}$, $B_j = \begin{bmatrix} 0 & P_j \\ P_j^* & 0 \end{bmatrix}$, $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$,

$P_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$, $P_2 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, $P_3 = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $j = 1, 2, 3$,

P_j^* denotes the conjugate matrix of the P_j^T, $U = (E_1, E_2, E_3, H_1, H_2, H_3)^T$, $\varepsilon(x_3)$, $\mu(x_3)$, and $\sigma(x_3)$ are unknown functions. $f(x_1, x_2, t)$ is known. Formulae (1.4) and (1.3) are usually called additional condition and source condition respectively.

1.2 Transformation and decomposition

Let $\tilde{U}(\nu_1, \nu_2, x_3, t) = \int_{R^2} U(x_1, x_2, x_3, t) \exp\Big[i \sum_{j=1}^{2} \nu_j (x_j - x_j^0)\Big] dx_1 dx_2$

$\tilde{f}(\nu_1, \nu_2, x_3, t) = \int_{R^2} f(x_1, x_2, x_3, t) \exp\Big[i \sum_{j=1}^{2} \nu_j (x_j - x_j^0)\Big] dx_1 dx_2$, here $i = \sqrt{-1}$

Let $q(z) = \sqrt[4]{\mu/\varepsilon}$, $z = \int_0^{x_3} \sqrt{\varepsilon\mu}\, dx_3$, $\tilde{U} = RV$, $V = (v_1, v_2, v_3, v_4, v_5, v_6)^T$

$$R = \begin{bmatrix} q & 0 & q & 0 & 0 & 0 \\ 0 & q & 0 & q & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1/q & 0 & -1/q & 0 & 0 \\ -1/q & 0 & 1/q & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad \begin{bmatrix} W_1 \\ W_2 \end{bmatrix} = \frac{1}{q(0)} \frac{\partial^2}{\partial t^2} \begin{bmatrix} v_1 \\ v_3 \end{bmatrix} \Big|_{\nu_1 = \nu_2 = 0}$$

$$\begin{bmatrix} W_3 \\ W_4 \end{bmatrix} = \frac{1}{q(0)} \frac{\partial^4}{\partial \nu_2^2 \partial t^2} \begin{bmatrix} v_1 \\ v_3 \end{bmatrix} \Big|_{\nu_1 = \nu_2 = 0}, \quad \begin{bmatrix} W_5 \\ W_6 \end{bmatrix} = \frac{1}{q(0)} \frac{\partial^2}{\partial t^2} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \Big|_{\nu_1 = 1, \nu_2 = 0}$$

$$\begin{bmatrix} W_1 \\ W_2 \end{bmatrix} = \begin{bmatrix} \alpha_1^{-2}(z) \\ \alpha_2^{-2}(z) \end{bmatrix} \delta''(t-z) + \begin{bmatrix} \alpha_1^{-1} \\ \alpha_2^{-1} \end{bmatrix} \delta'(t-z) + \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} \delta(t-z) + \begin{bmatrix} W_{10}(z,t) \\ W_{20}(z,t) \end{bmatrix}$$

$$\begin{bmatrix} W_3 \\ W_4 \end{bmatrix} = \begin{bmatrix} \alpha_3^{-1} \\ \alpha_4^{-1} \end{bmatrix} \delta'(t-z) + \begin{bmatrix} \alpha_3 \\ \alpha_4 \end{bmatrix} \delta(t-z) + \begin{bmatrix} W_{30} \\ W_{40} \end{bmatrix}$$