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THE PERIODIC BOUNDARY PROBLEM AND THE INITIAL VALUE PROBLEM FOR A NONLINEAR SYSTEM OF EQUATIONS OF CHANGING TYPE

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Abstract In this paper, we study the global existences of regular solutions, classical solutions and C^∞-solutions of the periodic boundary problem and the initial value problem for a nonlinear system of equations of changing type.
Key Words Changing type; regular solutions; classical solutions; C^∞-solutions; the periodic boundary problem; the initial value problem.
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In practice there are a lot of problems concerning higher-order equations of changing type [1–2]. But, there are only a few of papers studying nonlinear system of equations of changing type [3–4]. Now, we consider a general nonlinear system of equations of changing type

$$Lu = (K(t)u_t)_t + (-1)^{M+1}AD_x^{2M}u + \sum_{i=0}^{m}(-1)^{i+1}D_x^iF_i(u, D_xu, \cdots, D_x^mu) = f(x,t) \quad (1)$$

where $M > m \geq 0$ are integers, $u = (u_1, \cdots, u_N)^T, f = (f_1, \cdots, f_N)^T, p^i = (p_1^i, \cdots, p_N^i)^T = D_x^iu, F_i(p^0, \cdots, p^m) = \text{grad}_iF(p^0, \cdots, p^m) = \left(\frac{\partial F(p^0, \cdots, p^m)}{\partial p_1^i}, \cdots, \frac{\partial F(p^0, \cdots, p^m)}{\partial p_N^i}\right)^T$, $i = 0, 1, \cdots, m$; A is a $N \times N$ constant matrix, $K(t) = \text{diag}\{k_1(t), \cdots, k_N(t)\}, F(u, \cdots, D_x^mu)$ is a nonlinear function of vectors $u, D_xu, \cdots, D_x^mu$.

Assume that $K(t)$ and A satisfy the following conditions

$$\begin{cases} \text{(I)} & k_j(t) \in C^2[0,T],\ k_j > 0 \text{ for } t \in [0,t_0), k_j < 0 \text{ for } t \in (t_0,T], \\ & k_j'(t) \leq k_0 < 0, t \in [0,t_0], \quad j = 1, \cdots, N \\ \text{(II)} & A \text{ is a symmetric positively definite matrix,} \\ & (A\xi, \xi) \geq a_0|\xi|^2, \xi \in R^N, a_0 > 0 \end{cases} \quad (2)$$

It is obvious that, in the case $M = 1$, (1) is a second order system of elliptic type for $0 \leq t < t_0$, and is a second order system of hyperbolic type for $t_0 < t \leq T, t = t_0$

is its degenerate line, hence (1) is a nonlinear system of equations of mixed type. In the case $M > 1$, (1) is a system of hypoelliptic type for $0 \le t < t_0$, and is a system of ultrahyperbolic type for $t_0 < t \le T$, hence (1) is a nonlinear system of equations of changing type.

Assume that on the degenerate line $t = t_0$ the following normal connected conditions are satisfied

$$\lim_{t\to t_0-0}(k_j D_x^s D_t^{r+1}u_j, D_x^s D_t^{r+1}u_j)(t) = \lim_{t\to t_0+0}(k_j D_x^s D_t^{r+1}u_j, D_x^s D_t^{r+1}u_j)(t),$$
$$0 \le s + rM \le M,\ s = 0,1,\cdots,M;\ r = 0,1;\ j = 1,\cdots,N \tag{3}$$

In this paper, we shall denote by B^N the Cartesian product $\prod_{i=1}^{N} B_i$ of the Banach spaces $B_i = B(i = 1,\cdots,N)$.

1. The Periodic Boundary Problem

Let us consider the system (1) with the periodic boundary conditions

$$\begin{cases} u(x-D,t) = u(x+D,t), & x \in R, \quad t \in [0,T] \\ u(x,0) = \varphi(x), \varphi(x-D) = \varphi(x+D), & x \in R, D > 0 \end{cases} \tag{4}$$

To simplify notation we define

$$|u|_2^2 = |u|_{L_2}^2 = (u,u)(t) = \int_{-D}^{D}\sum_{j=1}^{N} u_j^2(x,t)dx$$

$$|u|_r^r = |u|_{L_r}^r = \int_{-D}^{D}\sum_{j=1}^{N} u_j^r(x,t)dx, \quad r \in [2,\infty)$$

$$\|u\|_2^2 = \|u\|_{L_2}^2 = [u,u](t) = \int_0^t (u,u)(\tau)d\tau$$

$$\|u\|_r^r = \|u\|_{L_r}^r = \int_0^t\int_{-D}^{D}\sum_{j=1}^{N} u_j^r(x,\tau)dxd\tau, \quad r \in [2,\infty)$$

$$|u|_{(n)}^2 = \sum_{j=1}^{N}|u_j|_{H^n}^2, \quad n \in [1,\infty)$$

Assume that F, f, φ satisfy the following conditions