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ON THE SOLVABILITY OF THE SAME NONLINEAR COMPOUND BOUNDARY VALUED PROBLEMS

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Abstract In this paper, we may make the following:

$$\begin{cases} |W(t)| = \phi(t), & t \in L \subset \partial D \\ \mathrm{Re}[a(t) - i \cdot b(t)]W(t) = \psi(t), & t \in M = \partial D - L \end{cases}$$

equal to searching for a positive solution of nonlinear singular integral equation. The solvability and discrete approximate solution of the singular integral equation have been studied.

Key Words Analytic function; nonlinear boundary valued problem; nonlinear singular integral equation.

Classification 35Q15, 45E00.

1. Introduction

Let D be a domain with boundary ∂D and $W(z)$ be an analytic function satisfying the following conditions

$$\begin{cases} |W(t)| = \phi(t), & t \in L \subset \partial D \quad (1) \\ \mathrm{Re}[a(t) - ib(t)]W(t) = 0, & t \in M = \partial D - L \quad (2) \end{cases}$$

where $\phi(t)$ is a real valued function on L and $a(t), b(t)$ are real valued functions on M, satisfying Hölder conditions, respectively. This nonlinear compound boundary valued problem has been studied by several authors[1,2,3].

In [1, 2], D is an upper half-plane, L is a bounded interval on real axis. In [3], D is a domain of unit circle $|z| < 1$, L is upper half-circumference $\mathrm{Im}\ z \geq 0$, M is under half-circumference.

In this paper, on the assumptions of [3], we remove condition (2) to a general Riemann-Hilbert problem. Namely, on $\partial D : |z| = 1$, $W(t)$ ($t = e^{is}$, $0 \le s \le 2\pi$) satisfying

$$|W(t)| = \phi(s), \qquad 0 \le s \le \pi \tag{1'}$$

$$\mathrm{Re}[a(s) - ib(s)]W(t) = \psi(s), \qquad \pi < s < 2\pi \tag{3}$$

where $\phi(s)$, $a(s)$, $b(s)$, $\psi(s)$ are real valued functions on ∂D, satisfying Hölder conditions, $\phi(s) \ne 0$ ($\forall s \in [0, \pi]$), $a^2(s) + b^2(s) \ne 0$ ($\forall s \in [\pi, 2\pi]$).

We may make the problems (1′)–(3) equal to searching for a positive solution of nonlinear singular integral equation. The solvability and discrete approximate solution of the singular integral equation have been studied (see [4–6]).

2. A Standard Solution of Problems (1′)–(3) on $\phi(s) \equiv 1$, $\psi(s) \ne 0$

Definition *$X(z)$ is called a standard solution of problems (1′)–(3) if it continues in $D + \partial D$ and $X(z) \ne 0$ for any z.*

Suppose that the standard solution of problems (1′)–(3) $X(z)$ exists, let

$$|X(t)| = \Phi^*(s) = \begin{cases} 1, & 0 \le s \le \pi \\ \phi^*(s), & \pi < s < 2\pi \end{cases} \tag{4}$$

from [7] we have

$$X(z) = \exp[S(\ln \Phi^*(s)) + ic], \qquad z \in D \tag{5}$$

where $S(\ln \Phi^*(s))$ is Schwarz's integral in which density is $\ln \Phi^*(s)$, c is any real constant.

Since $X(z)$ satisfies (3), it follows that

$$\mathrm{Re}[a(s) - ib(s)] \cdot \exp\left[\ln \phi^*(s) - \frac{i}{2\pi}\int_0^{2\pi} \ln \Phi^*(\sigma)\mathrm{ctg}\,\frac{\sigma - s}{2}d\sigma + ic\right] = \psi(s)$$

$$\pi < s < 2\pi$$

By simplifying, we have

$$\sqrt{a^2(s) + b^2(s)} \cdot \phi^*(s) \cdot \cos\left[\theta(s) - \frac{i}{2\pi}\int_0^{2\pi} \ln \Phi^*(\sigma)\mathrm{ctg}\,\frac{\sigma - s}{2}d\sigma + c\right] = \psi(s) \tag{6}$$

where $\theta(s)$ is an amplitude of complex valued function $a(s) - ib(s)$.

Since $X(z)$ is a standard solution, $\phi^*(s) > 0$ and continues. Hence there exists sufficiently small positive constant ε, such that the function

$$\zeta(s) = 1/\phi^*(s) - \varepsilon, \qquad \pi \le s \le 2\pi \tag{7}$$