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FORCED OSCILLATION FOR CERTAIN NONLINEAR DELAY PARABOLIC EQUATIONS*

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Abstract In this paper we investigate the forced oscillation of the solution of a class of nonlinear parabolic equations with continuous distributed deviating arguments.
Key Words Forced oscillation; parabolic equation; continuous distributed deviating arguments.
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In this paper we consider the following nonlinear parabolic equation with continuous distributed deviating arguments

$$\text{(E)} \qquad u_t = a(t)\Delta u - \int_a^b q(x,t,\xi)F[u(x,g(t,\xi))]d\sigma(\xi) + h(x,t),\ (x,t) \in \Omega \times R_+$$

where Ω is a bounded domain in R^n with piecewise smooth boundary $\partial\Omega$; $R_+ = [0,+\infty)$, $u = u(x,t)$, Δ is the Laplacian in R^n; $a(t) \in C(R_+, R_+)$, $q(x,t,\xi) \in C(\overline{\Omega} \times R_+ \times [a,b], R_+)$, $F(u) \in C(R,R)$; $g(t,\xi) \in C(R_+ \times [a,b], R)$, $g(t,\xi) \le t$, $\xi \in [a,b]$; $g(t,\xi)$ is a nondecreasing function with respect to t and ξ, respectively; and $\lim\limits_{t\to+\infty} \min\limits_{\xi\in[a,b]} \{g(t,\xi)\} = +\infty$; $\sigma(\xi) \in ([a,b], R)$ is nondecreasing in ξ; the forcing term $h(x,t) \in C(\overline{\Omega} \times R_+, R)$; the integral in (E) is Stieltjes integral.

We consider three kinds of boundary conditions:

$$(\mathrm{B}_1) \qquad u = \varphi, \quad (x,t) \in \partial\Omega \times R_+$$

$$(\mathrm{B}_2) \qquad \frac{\partial u}{\partial N} = \psi, \quad (x,t) \in \partial\Omega \times R_+$$

$$(\mathrm{B}_3) \qquad \frac{\partial u}{\partial N} + \mu u = 0, \quad (x,t) \in \partial\Omega \times R_+$$

where N is the unit exterior normal vector to $\partial\Omega$, φ and ψ are continuous functions on $\partial\Omega \times R_+$, and μ is a nonnegative continuous function on $\partial\Omega \times R_+$.

Some papers have been published concerning the oscillation theory of certain classes of delay parabolic equations. We mention here the work [1]–[4] and their references.

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Concerning forced oscillation of delay parabolic equations, only the work of N. Yoshida [4] is known. The case with discrete distributed deviating arguments all have been considered in those papers. However, it seems that very little is known about the work of the case with continuous distributed deviating arguments. We know only about the works of certain classes of ordinary differential equations with continuous distributed deviating arguments, e.g. see [5], [6]. In this paper we discuss the forced oscillation of the solution of the partial differential equation (E) with continuous distributed deviating arguments. Some oscillatory criteria are obtained for Equation (E) satisfying (B_1), (B_2) and (B_3), respectively.

Definition *The solution $u(x,t)$ of Equation* (E) *satisfying certain boundary condition is called oscillating in the domain $\Omega \times R_+$ if for each positive number τ there exists a point $(x_0, t_0) \in \Omega \times [\tau, +\infty)$ such that the condition $u(x_0, t_0) = 0$ holds.*

The following fact will be used:

The smallest eigenvalue α_0 of the Dirichlet problem

$$\begin{cases} \Delta u + \alpha u = 0 & \text{in } \Omega \\ u \mid_{\partial\Omega} = 0 \end{cases}$$

is positive and the corresponding eigenfunction $\Phi(x)$ is positive in Ω.

Lemma 1 *Let the following condition hold:*

(H_1) *$F(u)$ is a positive and convex function in the segment $(0, +\infty)$.*

If $u(x,t)$ is a positive solution of the problem (E), (B_1) *in the domain $\Omega \times [\tau, +\infty)$, $\tau \geq 0$, then the function*

$$X(t) = \left[\int_\Omega \Phi(x)dx\right]^{-1} \cdot \int_\Omega u(x,t)\Phi(x)dx \tag{1}$$

satisfies the inequality

$$(\mathrm{I}_1) \qquad X'(t) + \alpha_0 a(t) X(t) + \int_a^b Q(t,\xi) F[X(g(t,\xi))] d\sigma(\xi) \leq H(t)$$

where $Q(t,\xi) = \min_{x \in \overline{\Omega}} q(x,t,\xi)$,

$$H(t) = \left[\int_\Omega \Phi(x)dx\right]^{-1} \cdot \left[-a(t)\int_{\partial\Omega} \varphi \frac{\partial \Phi}{\partial N} dS + \int_\Omega h(x,t)\Phi(x)dx\right] \tag{2}$$

dS is an areal element of $\partial\Omega$.

Proof Suppose that $u(x,t)$ is a positive solution of the problem (E), (B_1) in $\Omega \times [\tau, +\infty)$, $\tau \geq 0$. Note that

$$\lim_{t\to+\infty} \min_{\xi\in[a,b]} \{g(t,\xi)\} = +\infty$$

so there exists a $t_1 \geq \tau$ such that

$$u(x, g(t,\xi)) > 0, \quad t \geq t_1,\ \xi \in [a,b]$$