J.Partial Diff. Eqs. 8(1995), 351–370
©International Academic Publishers Vol.8, No.4

GLOBAL PERTURBATION OF THE RIEMANN PROBLEM FOR THE SYSTEM OF COMPRESSIBLE FLOW THROUGH POROUS MEDIA *

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(Received Oct. 22, 1993; revised June 3, 1994)

Abstract In this paper we consider the unperturbated and perturbated Riemann problem for the damped quasilinear hyperbolic system

$$\begin{cases} v_t - u_x = 0 \\ u_t + p(v)_x = -\alpha u, \quad \alpha > 0,\ p'(v) < 0 \end{cases}$$

with initial structure of two rarefaction waves or one rarefaction wave plus one shock wave. Under certain restrictions, it admits a unique global discontinuous solution in a class of piecewise continuous and piecewise smooth functions and keeps the initial structure. Moreover, the shock strength is found decaying exponentially due to damping for the later case.
Key Words Hyperbolic; Riemann problem; perturbation; global structure.
Classification 35L45.

1. Introduction

Consider the following quasilinear hyperbolic system

$$\begin{cases} v_t - u_x = 0 \\ u_t + p(v)_x = -\alpha u, \quad \alpha > 0,\ p'(v) < 0 \end{cases} \tag{1.1}$$

which models the compressible flow through porous media where u is the velocity, $v > 0$ is the specific volume and $p(v)$ is the pressure.

We study the discontinuous initial value problem of (1.1), namely

$$(u(0,x), v(0,x)) = \begin{cases} (u_-(x), v_-(x)) & \text{for } x < 0 \\ (u_+(x), v_+(x)) & \text{for } x > 0 \end{cases} \tag{1.2}$$

where $u_\mp(x)$ and $v_\mp(x)$ are smooth functions such that $\lim\limits_{x\to 0}(u_\mp(x), v_\mp(x)) = (u^\mp, v^\mp)$ with $v^\mp > 0$. This kind of discontinuous initial value problem may be called as perturbated Riemann problem.

*The project partially supported by National Natural Science Foundation of China.

For the system without the damping term, i.e.

$$\begin{cases} v_t - u_x = 0 \\ u_t + p(v)_x = 0, \quad p'(v) < 0 \end{cases} \tag{1.3}$$

the global perturbation of the Riemann problem has been studied in [1], where it has been proved under certain restriction on the initial data (1.2) that the problem (1.3) (1.2) admits a unique global solution in a class of piecewise continuous and piecewise smooth functions, and the solution has a global structure similar to that of the corresponding Riemann problem with Riemann data (u^-, v^-) and (u^+, v^+). The Riemann problem for (1.3) has been studied very well. (See [2]–[5], etc.)

For the system (1.1) with damping term, however, the Riemann problem is much more complicated since there is no self-similar solution anymore, and has not been well studied in the literature although the problem is significant in the qualitative theory for inhomogeneous hyperbolic system and in the applications as well. Moreover, it is expected that the large time behavior of solutions for (1.1) exhibits an interesting nonlinear diffusive phenomena (see [6] for detail).

We investigate the perturbated Riemann problem (1.1) (1.2) and construct the globally defined piecewise continuous and piecewise smooth solutions with showing the qualitative behavior and the global structure which is similar to that of the corresponding Riemann problem with Riemann data (u^-, v^-) and (u^+, v^+). For simplicity, we take a typical form of the state function $p(v)$ instead of a general form, namely, $p(v) = av^{-\gamma}$ with $a > 0$ and $1 \leq \gamma < 3$, which is the state function for polytropic gas. According to the different relative location of the two states (u^-, v^-) and (u^+, v^+) in the phase plane, we study the problem in separate papers. The case in which the two states are connected by a backward shock curve and a forward shock curve subsequently have been discussed in [7] and [8].

In the present paper we discuss two cases, in the first case the two states (u^-, v^-) and (u^+, v^+) are connected by a backward rarefaction wave curve and a forward shock wave curve, in the second case by a backward rarefaction wave curve and a forward rarefaction wave curve subsequently. For the later case a discussion for $p(v) = av^{-\gamma}$ with $0 < \gamma \leq 1$ can be found in [9]. The case in which these two states are connected by a backward shock curve and a forward rarefaction wave curve can be handled in a similar way as in the present paper.

Under certain restriction on the perturbation in $(u_\mp(x), v_\mp(x))$, we prove that the problem (1.1), (1.2) admits a unique global discontinuous solution on $t \geq 0$ in a class of piecewise continuous and piecewise smooth functions which contains a backward rarefaction wave and a forward shock wave for the first case. The shock does not disappear for any finite time, and does disappear with the strength decay exponentially fast when time t tends to infinity. One of the most striking features of the damped system lies in this exponential decaying of shock strength. We prove the result for unperturbated Riemann data in Section 2 and the result for perturbated data in Section 3. Section 4 is devoted to the second case. For the sake of clarity, detailed analysis is