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A QUASISTEADY STEFAN PROBLEM WITH CURVATURE CORRECTION AND KINETIC UNDERCOOLING*

Yu Wanghui
(Department of Mathematics, Suzhou University, Suzhou 215006, China)
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Abstract A quasisteady Stefan problem with curvature correction and kinetic undercooling is considered. It is a problem with phase transition, in which not only the Stefan condition, but also the curvature correction and kinetic undercooling effects hold on the free boundary, and in phase regions elliptic equations are satisfied by the unknown temperature at each time. The existence and uniqueness of a local classical solution of this problem are obtained.
Key Words Stefan problem; curvature correction; kinetic undercooling.
Classification 35R35, 35B45.

1. Introduction

Let Ω be a bounded domain in $\mathbf{R}^N$ and Γ^0 be a $(N-1)$-dimensional single-connected closed hypersurface in Ω. Our problem is to determine a function $u(x,t):\Omega\times[0,T]\to\mathbf{R}^1$ and a free boundary $\Gamma\equiv\bigcup_{0\le t\le T}(\Gamma_t\times\{t\})$, where $\Gamma_t(0\le t\le T)$ are $(N-1)$-dimensional single-connected closed hypersurfaces in Ω, such that

$$\begin{cases}\Delta u^i(\cdot,t)=0,\ \text{in }\Omega^i(t),\quad 0\le t\le T\quad i=1,2 & (1.1)\\ \dfrac{\partial u^1}{\partial n}=0\quad\text{on }\partial\Omega\times[0,T] & (1.2)\\ u^1=u^2=-\widehat{\alpha}\kappa-\widehat{\beta}V\quad\text{on }\Gamma_t,\quad 0\le t\le T & (1.3)\\ \dfrac{\partial u^1}{\partial\nu}-\dfrac{\partial u^2}{\partial\nu}=V\quad\text{on }\Gamma_t,\quad 0\le t\le T & (1.4)\\ \Gamma_0=\Gamma^0 & (1.5)\end{cases}$$

where, $u^i\equiv u$ in $\overline{\Omega^i(t)}\times[0,T]$, $i=1,2$; $\Omega^i(t)$ is the domain bounded by Γ_t for $i=1$, or by Γ_t and $\partial\Omega$ for $i=2$; n and ν are the unit outward normal vectors of Ω and $\Omega^1(t)$ respectively; κ is the mean curvature of Γ_t which takes positive value when $\Omega^1(t)$ protrudes into $\Omega^2(t)$; V is the velocity of Γ_t in the direction of ν; $\widehat{\alpha}$ and $\widehat{\beta}$ are positive constants.

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If (1.1) is replaced by the heat equation

$$u_t^i - \Delta u^i = 0 \text{ in } \bigcup_{0\le t\le T} (\Omega^i(t)\times\{t\}), \quad i=1,2 \tag{1.6}$$

the system (1.2)–(1.6) is called the Stefan problem with the curvature correction and kinetic undercooling. This problem has been studied by many mathematicians. Chen, X. and Reitch, F. ([1]) established the existence and uniqueness of a local classical solution for $\widehat{\beta} > 0$ and $\widehat{\alpha} > 0$. Radkevich, E. ([2]) proved the existence of a local classical solution for $\widehat{\beta} \ge 0$ and $\widehat{\alpha} > 0$. Meimanov, A.M. ([3]) considered the problem in spherical symmetric case and obtained the existence and uniqueness of a global classical solution when $\widehat{\beta} > 0$ and $\widehat{\alpha} > 0$, and the nonexistence of a global classical solution when $\widehat{\beta} = 0$ and $\widehat{\alpha} > 0$. Liu, Z. ([4]) prove the existence and uniqueness of a global classical solution for $\widehat{\beta} > 0$ and $\widehat{\alpha} > 0$ in the two-dimensional axis symmetric case and he also got sufficient conditions for the convexness of the free boundary. And Luckhaus, S. ([5] [6]) set up a weak formulation for $\widehat{\beta} = 0$ and $\widehat{\alpha} > 0$, and proved the existence of a global weak solution of this problem.

For the quasisteady Stefan problem with curvature correction and kinetic undercooling, that is, the system (1.1)–(1.5), less results have been obtained. When $\widehat{\beta} = 0$, $\widehat{\alpha} > 0$ and $\partial\Omega$ being a chart of a Lipschitz function defined in $\mathbf{R}^1 (N = 2)$, Duchon, J. and Robert, R. ([7]) proved the existence and uniqueness of a local classical solution. When $\widehat{\beta} = 0$, $\widehat{\alpha} > 0$ and Ω being a bounded domain in $\mathbf{R}^2$, Chen, X. ([8]) established the existence of a local weak solution, and he also obtained the existence and asymptotic behaviors of a global weak solution when $\widehat{\beta} = 0$, $\widehat{\alpha} > 0$ and $\Omega = \mathbf{R}^2$.

In this paper we consider the system (1.1)–(1.5) in the case that $\widehat{\beta} > 0$ and $\widehat{\alpha} > 0$. We shall use the idea in [1] to formulate (1.3) as a parabolic equation on some smooth manifold. Our main results are the existence and uniqueness of a local classical solution. Without loss of generality, we might as well suppose $\widehat{\alpha} = \widehat{\beta} = 1$ in the sequel.

Now we state our main results as follows:

Theorem 1.1 *Assume that $\Gamma^0 \in C^{5+\alpha}$ and $\partial\Omega \in C^{3+\alpha}$ $(0<\alpha<1)$, then there exists a positive constant T, depending only on Γ^0 and Ω, such that, the system (1.1)–(1.5) has an unique classical solution $\{u,\Gamma\}$, satisfying*

$$u \in C^{2+\alpha,\frac{\alpha}{2}}(\overline{Q_T^1}) \cap C^{2+\alpha,\frac{\alpha}{2}}(\overline{Q_T^2}) \cap C(\overline{\Omega}_T) \tag{1.7}$$

$$u(\cdot,t) \in C^{3+\alpha}(\overline{\Omega^1(t)}) \cap C^{3+\alpha}(\overline{\Omega^2(t)}), \quad 0\le t\le T \tag{1.8}$$

and

$$\Gamma \in C_{x,t}^{4+\alpha,\frac{2+\alpha}{2}}$$

where $\Omega_T \equiv \Omega\times(0,T)$, $Q_T^i \equiv \bigcup_{0\le t\le T}(\Omega^i(t)\times\{t\})$ $(i=1,2)$

$$C_{x,t}^{2+\alpha,\frac{\alpha}{2}} \equiv \{v \mid v, v_x, v_{xx} \in C^{\alpha,\frac{\alpha}{2}}\} \tag{1.9}$$