J.Partial Diff. Eqs. **9**(1996), 71–83
©International Academic Publishers Vol.9, No.1

ON THE SOLUTIONS OF THE COUPLED NONLINEAR PARABOLIC EQUATIONS *

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(Received April 20, 1992; revised July 27, 1995)

Abstract By means of the fixed point technique and integral estimation method, we study the solutions of periodic boundary value problem and initial value problem for the coupled nonlinear parabolic equations. The global classical solutions of the mentioned problems are shown to exists.
Key Words Nonlinear parabolic equation, existence and uniqueness global solution
Classification 35Q20.

1. Introduction and Preliminaries

The high resolution numerical simulation of plasma turbulence driven by ion temperature gradients in the presence of magnetic field inhomogeneities was performed in [1] with special attention to the behavior of the anomalous ion energy flux. The pressure gradient evolution is treated consistently with energy transport, allowing for the study of the saturated state in situations of relevance to Tokamak plasmas. Under some assumptions, the dynamical equations of plasma are reduced to the following coupled nonlinear parabolic equations [1]:

$$\Phi_t - \Delta\Phi_t + \alpha\Delta^2\Phi + J(\Delta\Phi, \Phi) + (1-\gamma)\Phi_y - \gamma\Psi_y = 0, \quad (x,y) \in \mathbf{R}^2,\ \mathbf{t} > \mathbf{0} \quad (1)$$

$$\Psi_t - \beta\Delta\Psi + J(\Phi, \Psi) + \gamma\Phi_y + \delta\Psi_y = f(x,y,t),$$
$$(x,y) \in \mathbf{R}^2,\ \mathbf{t} > \mathbf{0},\ \alpha > \mathbf{0},\ \beta > \mathbf{0} \quad (2)$$

with the periodic boundary value problem (PBVP)

$$\Phi(x+2D, y, t) = \Phi(x, y+2D, t) = \Phi(x,y,t), \quad (x,y) \in \mathbf{R}^2,\ \mathbf{t} > \mathbf{0} \quad (3)$$
$$\Psi(x+2D, y, t) = \Psi(x, y+2D, t) = \Psi(x,y,t), \quad (x,y) \in \mathbf{R}^2,\ \mathbf{t} > \mathbf{0} \quad (4)$$
$$\Phi(x,y,0) = \Phi_0(x,y), \quad (x,y) \in \mathbf{R}^2 \quad (5)$$
$$\Psi(x,y,0) = \Psi_0(x,y), \quad (x,y) \in \mathbf{R}^2 \quad (6)$$

*The project supported by National Natural Science Foundation of China.

where the initial functions satisfy the periodic condition

$$\Phi_0(x+2D,y)=\Phi_0(x,y+2D)=\Phi_0(x,y),\quad (x,y)\in\mathbf{R}^2 \tag{7}$$

$$\Psi_0(x+2D,y)=\Psi_0(x,y+2D)=\Psi_0(x,y),\quad (x,y)\in\mathbf{R}^2 \tag{8}$$

and the initial value problem (IVP)

$$\Phi(x,y,0)=\Phi_0(x,y),\quad (x,y)\in\mathbf{R}^2 \tag{9}$$

$$\Psi(x,y,0)=\Psi_0(x,y),\quad (x,y)\in\mathbf{R}^2 \tag{10}$$

In Equations (1) and (2), Φ and Ψ are the physical functions corresponding to the density and pressure, respectively. Δ is two-dimensional Laplace operator; $J(a,b)$ is the determinant of Jacobi matrix, namely, $J(a,b)=a_xb_y-a_yb_x$; α and β are positive constants, γ and δ are real constants; $f(x,y,t)$ is a suitable energy source. In Conditions (3–4) and (7–8), D is a positive constant.

We remark that Guo Boling paid attention to the similar coupled nonlinear evolution equations [2]

$$\Delta\Psi_t+J(\Psi,\Delta\Psi)-\Delta^2\Psi+\alpha\theta_x=0,\quad (x,y)\in\mathbf{R}^2,\ \mathbf{t>0} \tag{11}$$

$$\theta_t+J(\Psi,\theta)-\beta\Delta\theta=0,\quad (x,y)\in\mathbf{R}^2,\ \mathbf{t>0},\ \alpha>0,\ \beta>0 \tag{12}$$

Zhou Yulin et al paid attention to the nonlinear evolution equation [3]

$$\Psi_t-\Delta\Psi_t+J(\Delta\Psi,\Psi)+\alpha\Delta\Psi_x+\beta\Delta\Psi_y+f(\Psi)_x+g(\Psi)_y=h(\Psi) \tag{13}$$

but there has been no contribution to this type of system up to now.

Let T be a finite positive constant. Denote by Ω and Ω_T the domains

$$\Omega=\{(x,y)\in\mathbf{R}^2 \mid \mathbf{0\le x,\ y\le 2D}\}$$

$$\Omega_T=\{(x,y,t)\in R^3 \mid 0\le x,\ y\le 2D,\ 0\le t\le T\}$$

To simplify the notations, we will denote by C all the positive constants appeared in the paper afterward, which do not depend on the solutions or their derivatives of any order, nor will they depend on D.

For any $f(x,y),g(x,y)\in C^1(\Omega)$, if they satisfy the periodic conditions (7) and (8), then we have the relation

$$\int_\Omega fJ(f,g)dxdy=0,\quad \int_\Omega gJ(f,g)dxdy=0$$

2. *A Priori* Estimate

In this section, we establish a series of integral estimates of the solutions of problem (1)–(6).