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GLOBAL EXISTENCE FOR A CLASS OF NON-FICKIAN POLYMER-PENETRANT SYSTEMS*

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Abstract This paper deals with a class of strongly coupled and highly degenerate nonlinear parabolic systems, which arises from a model describing non-Fickian diffusion of penetrant into glassy polymers. By means of a fixed point argument and *a priori* estimates, we establish the global existence and uniqueness for the systems.

Key Words *A priori* estimates; nonlocal PDE; global existence.
Classifications 35K20, 35K55.

1. Introduction

The diffusion in swollen media is most commonly described by Fick's law

$$J(x,t) = -D(c)\frac{\partial c}{\partial x} \tag{1.1}$$

where x is the spatial variable, c is the concentration of the penetrant, J is the fluid mass intake, or *flux*, and $D(c)$ is the diffusion coefficient. Fick's law is supplemented by the conservation of mass

$$\frac{\partial c(x,t)}{\partial t} = -\frac{\partial J(x,t)}{\partial x} \tag{1.2}$$

Swollen rubbery polymers obey the (concentration-dependent) Fick's law. However, it cannot adequately describe the diffusion in glassy polymers. In fact, the diffusion in glassy polymers causes micro-mechanical changes in the polymer structure and this results in a non-Fickian diffusion.

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Suppose that one face of a slab of dry polymer is in contact with fluid (penetrant); then the fluid will penetrate into the slab through this face. Cohen and White [1] proposed the following model to describe the diffusion of the penetrant in the glassy polymer. They take

$$J = -D(c)\frac{\partial c}{\partial x} - E(c)\frac{\partial \sigma}{\partial x} + Mc \tag{1.3}$$

where σ is the stress; $E(c)\dfrac{\partial \sigma}{\partial x}$ is the component of the flux representing the viscoelastic effect with the appropriate proportionality factor E, and Mc represents convective effects. Using some viscoelastic models which relate σ to the strain ε and imposing some relationship between ε and c, they derive the relation (for more details see [1])

$$\frac{\partial \sigma}{\partial t} + \beta(c)\sigma = \rho(c)c + \gamma(c)\frac{\partial c}{\partial t} \tag{1.4}$$

where $\beta(c)$, $\rho(c)$, $\gamma(c)$ are smooth functions of c (see also [Chapter 4, 2] for more details).

They system (1.2), (1.3) and (1.4) needs to be supplemented with initial and boundary conditions. For one dimensional case, the complete problem is

$$c_t = (Dc_x + E(c)\sigma_x)_x - (Mc)_x \quad \text{for } 0 < x < 1,\ t > 0 \tag{1.5}$$

$$\sigma_t + \beta(c)\sigma = \rho(c)c + \gamma(c)c_t \quad \text{for } 0 < x < 1,\ t > 0 \tag{1.6}$$

$$c(0,t) = \theta > 0, \quad c(1,t) = 1 > \theta \quad \text{for } t > 0 \tag{1.7}$$

$$c(x,0) = c_0(x) \geq 0 \quad \text{for } 0 < x < 1 \tag{1.8}$$

$$\sigma(x,0) = \sigma_0(x) \quad \text{for } 0 < x < 1 \tag{1.9}$$

where θ is a constant.

Definition *We refer to the system* (1.5)-(1.9) *as Problem* 1.

From the mathematical point of view Problem 1 is a strongly coupled (second derivatives are coupled) and highly degenerate nonlinear parabolic system, whose "diffusion matrix" has rank one only. This problem was studied by H. Amann as a special case of the more general highly degenerate parabolic systems with n spatial variables in [3, 4] where he established the local existence and uniqueness of the solution by using semigroup theory. In this paper we shall use a different method and obtain the global existence and uniqueness for Problem 1.

In view of particular choices of functions $E(c)$, $\beta(c)$, $\rho(c)$, $\gamma(c)$ based on the physical considerations (see, for example, [1] [2]), we shall assume, throughout this paper, that they are smooth enough so that they have the required different ability, and

$$E(0) = 0; \quad E(c) > 0; \quad 0 \leq E'(c) \leq E_1; \quad |E''(c)| \leq E_1 \quad \text{for } c \geq 0 \tag{1.10}$$