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The Implication of Local Thin Plate Splines for Solving Nonlinear Mixed Integro-Differential Equations Based on the Galerkin Scheme

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Abstract. In this article, we investigate the construction of a computational method for solving nonlinear mixed Volterra-Fredholm integro-differential equations of the second kind. The method firstly converts these types of integro-differential equations to a class of nonlinear integral equations and then utilizes the locally supported thin plate splines as a basis in the discrete Galerkin method to estimate the solution. The local thin plate splines are known as a type of the free shape parameter radial basis functions constructed on a small set of nodes in the support domain of any node which establish a stable technique to approximate an unknown function. The presented method in comparison with the method based on the globally supported thin plate splines for solving integral equations is well-conditioned and uses much less computer memory. Moreover, the algorithm of the presented approach is attractive and easy to implement on computers. The numerical method developed in the current paper does not require any cell structures, so it is meshless. Finally, numerical examples are considered to demonstrate the validity and efficiency of the new method.

AMS subject classifications: 45G10, 45J05, 65R20

Key words: Mixed integro-differential equation, nonlinear integral equation, discrete Galerkin method, local thin plate spline, meshless method.

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1. Introduction

In the early 1900, Vito Volterra studied the integro-differential equations in modeling the phenomenon of population growth and the effect of inheritance. These types of equations can be given as a class of integral equations including the derivatives of the

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unknown function in which the derivatives are sometimes in the integral operator and sometimes out of it. There exist significant applications of integro-differential equations in both Fredholm and Volterra types, such as natural sciences, engineering, mathematical modeling of spatiotemporal developments and epidemic modeling [51, 57]. Integro-differential equations also occur as reformulations from some boundary value problems arising in various branches of applied science; for example, heat transfer, emission phenomena, neutron diffusion and so on [11, 18, 57].

Consider nonlinear mixed Volterra-Fredholm integro-differential equations of the second kind as follows [25]:

$$\frac{du}{dx} = f(x) + \lambda_1 \int_a^b K_1(x, y, u(y))dy + \lambda_2 \int_a^x K_2(x, y, u(y))dy, \qquad u(0) = u_0, \quad (1.1)$$

where $x, y \in [a, b]$, the function $f(x)$ is given, the unknown function $u(x)$ must be determined, $|\lambda_1| + |\lambda_2| \neq 0$, $\lambda_1, \lambda_2 \in \mathbb{R}$, and the kernels K_1 and K_2 are assumed to be known continuous functions on $[a, b]^2 \times \mathbb{R}$ and satisfy the Lipschitz condition that is, there exist $M_1, M_2 \geq 0$ such that:

$$\big|K_i(x, y, z) - K_i(x, y, z')\big| \leq M_i|z - z'|, \qquad z, z' \in \mathbb{R}, \qquad i = 1, 2.$$

Several numerical methods have been applied to find the approximate solution of integro-differential equations. The wavelet-Galerkin method [9, 33] has been used to solve nonlinear integro-differential equations. Zhao et al. [64] give sixth-order compact finite difference formula for solving integro-differential equations with different boundary conditions. The paper [47] has presented the single-term Walsh series method for the solution of Volterra integro-differential equations. He's variational iteration and Adomian decomposition [57] methods as semi-analytical approaches have been introduced to solve different types of integro-differential equations. Hybrid Legendre polynomials and block-pulse functions have been utilized to obtain the approximate solution of integro-differential equations in the works [19, 28]. Authors of [13] have studied the differential conversion method for the linear first order Fredholm integro-differential equations. The improved Legendre method [44] has been developed for a class of integro-differential equations which involves a population model.

In recent years, the meshfree methods based on the radial basis functions (RBFs) have significant applications in different problems of computational mathematics. The RBF–based schemes have been applied for solving Fredholm integral equations on non-rectangular domains with sufficiently smooth kernels [1, 3]. The discrete collocation method by combining RBFs has been used to estimate the solution of Volterra integral equations [4]. The meshless product integration (MPI) method [2] has been proposed to solve linear weakly singular integral equations.

An RBF–Galerkin scheme has been applied to obtain the numerical solution of singular boundary integral equations [5] and Hammerstein integral equations [7]. Authors of [23, 24] have investigated a domain RBF collocation method and a boundary RBF collocation method to solve fractional diffusion models. The RBFs as a basis in the