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Mathematical Modeling and Simulation of Antibubble Dynamics

Junxiang Yang[1], Yibao Li[2], Darae Jeong[3] and Junseok Kim[1,*]

[1] *Department of Mathematics, Korea University, Seoul 02841, Republic of Korea*
[2] *School of Mathematics and Statistics, Xi'an Jiaotong University, Xi'an 710049, China*
[3] *Department of Mathematics, Kangwon National University, Gangwon-do 24341, Republic of Korea*

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Abstract. In this study, we propose a mathematical model and perform numerical simulations for the antibubble dynamics. An antibubble is a droplet of liquid surrounded by a thin film of a lighter liquid, which is also in a heavier surrounding fluid. The model is based on a phase-field method using a conservative Allen–Cahn equation with a space-time dependent Lagrange multiplier and a modified Navier–Stokes equation. In this model, the inner fluid, middle fluid and outer fluid locate in specific diffusive layer regions according to specific phase filed (order parameter) values. If we represent the antibubble with conventional binary or ternary phase-field models, then it is difficult to have stable thin film. However, the proposed approach can prevent nonphysical breakup of fluid film during the simulation. Various numerical tests are performed to verify the efficiency of the proposed model.

AMS subject classifications: 35Q35, 76T10, 81T80

Key words: Antibubble, conservative Allen–Cahn equation, Navier–Stokes equation.

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1. Introduction

In this paper, we propose a mathematical model for the antibubble dynamics and perform several numerical simulations. An antibubble is a droplet of liquid surrounded by a thin film of a lighter liquid, which is also in a heavier surrounding fluid, see Fig. 1 for the antibubble formation.

The terminology of antibubble first occurred in [2] and the antibubble can be generated in many fluids, such as diesel oil [3], soapy water, Belgian beer [4], etc. Unlike a single bubble system, the antibubble system consists of three fluid components: inner core fluid (fluid 1), middle fluid film (fluid 2) and outer ambient fluid (fluid 3). Usually,

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[*] Corresponding author. *Email address:* `cfdkim@korea.ac.kr` (J. Kim)

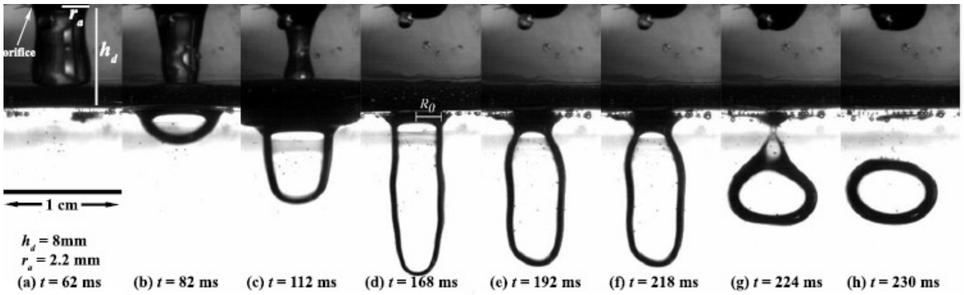

Figure 1: Formation of an antibubble. Reprinted from [1], with permission from IOP Publishing LTD.

the inner fluid and the outer fluid are the same. Because of the special fluid structure, the form of antibubble can be applied in some scientific, industrial and medical fields, including mass transfer of air [5], fluid transportation in a magnetic field [6], emulsion formation [7]. Furthermore, if the inner fluid is different from the outer fluid, then the antibubble structure can be extended for liquid drag delivery [8].

Recently, some researchers have conducted many theoretical and experimental studies on the antibubble. Scheid et al. [9] proposed a dynamic model for explaining the effect of surface shear viscosity on the drainage of air film. Sob'yanin [10] proposed a theory of antibubble collapse to account for the motion of air rim which can not be well explained by potential flow. Zou et al. [11] investigated the collapse of an antibubble, they found that the small bubbles are generated along the rim of air film during the collapse process. Kim and Stone [1] experimentally studied the dynamics of antibubble and provided the optimal condition for antibubble formation. Bai et al. [12] proposed a new experimental method for antibubble formation. However, to the best knowledge of authors, there are few numerical investigations of antibubble. Kim [13] performed a simulation of antibubble raising by using ternary Cahn–Hilliard–Navier–Stokes system, while the middle fluid film is set thick and the stability can not be satisfied in a long time.

In this paper, we propose a mathematical model for antibubble simulation using phase-filed method. Phase-field method is a popular issue in multiphase flow field and large amounts of researches have been studied by some scholars [14–18]. The proposed model is based on the conservative Allen–Cahn equation [19], which can avoid mass spreading to the bulk phase. In this model, the inner fluid locates in the region where the order parameter $\phi \geq 0$, the middle fluid film occupies the region where $-0.9 < \phi < 0$ and the rest of the region is filled with the outer ambient fluid. This approach can naturally keep a stable thin fluid film.

The remaining parts of this paper are organized as follows. In Section 2, we give the mathematical model. The numerical solution of the proposed model is presented in Section 3. To validate the new proposed model, various numerical tests are performed in Section 4. In Section 5, conclusions are drawn.