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High-Order Methods for Exotic Options and Greeks Under Regime-Switching Jump-Diffusion Models

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Abstract. This paper aims to develop high-order numerical methods for solving the system partial differential equations (PDEs) and partial integro-differential equations (PIDEs) arising in exotic option pricing under regime-switching models and regime-switching jump-diffusion models, respectively. Using cubic Hermite polynomials, the high-order collocation methods are proposed to solve the system PDEs and PIDEs. This collocation scheme has the second-order convergence rates in time and fourth-order rates in space. The computation of the Greeks for the options is also studied. Numerical examples are carried out to verify the high-order convergence and show the efficiency for computing the Greeks.

AMS subject classifications: 91G20, 91G60, 91G80

Key words: Option pricing, Greeks, exotic options, Asian options, lookback options, high-order methods.

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1. Introduction

The regime-switching model for the dynamics of stock price is first introduced by Hamilton [9,10]. The pricing of standard European or American options under regime-switching models has been well studied in the literature (see e.g., Bollen [2], Duan et al. [6], Khaliq and Liu [14], Yuen and Yang [27,28], Liu [16,17], Liu and Zhao [18], Ma and Zhou [20], Zhou et al. [29], Xing and Ma [26]). The extension to the regime-switching jump-diffusion models in regards to the standard options is studied by e.g., Jiang et al. [13], Ma et al. [21].

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The pricing of exotic options under regime-switching models has also been received attention. Boyle and Draviam [3] derive the PDEs for pricing lookback options and arithmetic Asian options under regime-switching models and develop an implicit scheme with an exponential interpolation to solve the PDEs. Boyle-Draviam's method is correct and efficient, however the rates of convergence (one order for both time and space) are not high-order as shown by a numerical example. Ma and Zhou [19] develop the moving mesh methods for Asian options under the regime-switching models. For the Asian options under regime-switching jump-diffusion models, Dang et al. [4] construct a sequence of PDEs by treating the integral term in the PIDEs explicitly and then use the Crank-Nicolson time-stepping schemes and cubic finite element methods to solve the sequence PDEs. The possible disadvantages of the approach are that it needs to solve a sequence of the PDEs and the calculation of the Greeks is not efficient as the first-order derivatives of the numerical solutions at the grid points are discontinuous.

In this paper, we develop a high-order method using the piecewise cubic Hermite polynomial approximation for solving the system PDEs and PIDEs arising in the exotic option pricing under regime-switching models and regime-switching jump-diffusion models. Following Huang and Russell [12], Ma, Huang and Russell [22], we use the piecewise cubic Hermite polynomials to construct the fully discretized high-order scheme. The scheme has second-order convergence rates in time and fourth-order rates in space. This high-order convergence in space is verified by numerical examples. Since the first-order derivatives of the numerical solutions are continuous for the piecewise cubic Hermite polynomial approximation, it is efficient to directly compute the Greeks Δ using the high-order methods, which is verified by the numerical examples.

The remaining parts of the paper are arranged as follows: In Section 2, we present the pricing PDEs for exotic options under regime-switching models and regime-switching jump-diffusion models and derive the high-order scheme and calculate the Greeks; In Section 3, we carry out numerical examples to compare with the existing methods in the literature and verify the high-order convergence; In the final section, we give the conclusions.

2. High-order methods for exotic options

Let $(\Omega, \mathcal{F}, \mathcal{Q})$ be a complete probability space, $\mathcal{Q}$ is risk-neutral measure. S_t denote the underlying asset prices. Our main models are as follows:

Table 1: The models.

Models	Type	Process
Model 1	Regime-switching model	$\frac{dS_t}{S_t} = r\left(Z(t)\right)dt + \sigma\left(Z(t)\right)dW_t$
Model 2	Regime-switching jump-diffusion model	$\frac{dS_t}{S_t} = \left[r\left(Z(t)\right) - \omega\left(Z(t)\right)\kappa\left(Z(t)\right)\right]dt + \sigma\left(Z(t)\right)dW_t + \left[\eta\left(Z(t)\right) - 1\right]d\aleph_t$