Numer. Math. Theor. Meth. Appl.
doi: 10.4208/nmtma.OA-2019-0015

Vol. **13**, No. 3, pp. 665-688
August 2020

Multidomain Legendre-Galerkin Least-Squares Method for Linear Differential Equations with Variable Coefficients

Yonghui Qin[2,3], Yanping Chen[1,*], Yunqing Huang[2] and Heping Ma[4]

[1] *School of Mathematical Science, South China Normal University, Guangzhou 520631, Guangdong, China*
[2] *School of Mathematics and Computational Science, Hunan Key Laboratory for Computation and Simulation in Science and Engineering, Xiangtan University, Xiangtan 411105, Hunan, China*
[3] *College of Mathematics and Computing Science, Guangxi Colleges and Universities Key Laboratory of Data Analysis and Computation, Guangxi Key Laboratory of Cryptography and Information Security, Guilin University of Electronic Technology, Guilin 541004, Guangxi, China*
[4] *Department of Mathematics, Shanghai University, Shanghai 200444, China*

Received 28 January 2019; Accepted (in revised version) 13 December 2019

Abstract. The multidomain Legendre-Galerkin least-squares method is developed for solving linear differential problems with variable coefficients. By introducing a flux, the original differential equation is rewritten into an equivalent first-order system, and the Legendre Galerkin is applied to the discrete form of the corresponding least squares function. The proposed scheme is based on the Legendre-Galerkin method, and the Legendre/Chebyshev-Gauss-Lobatto collocation method is used to deal with the variable coefficients and the right hand side terms. The coercivity and continuity of the method are proved and the optimal error estimate in H^1-norm is obtained. Numerical examples are given to validate the efficiency and spectral accuracy of our scheme. Our scheme is also applied in the numerical solutions of the parabolic problems with discontinuous coefficients and the two-dimensional elliptic problems with piecewise constant coefficients, respectively.

AMS subject classifications: 35F25, 65M70, 65L06

Key words: Variable coefficient, Legendre Galerkin, Legendre/Chebyshev-Gauss-Lobatto, least squares.

*Corresponding author. *Email addresses:* `yanpingchen@scnu.edu.cn` (Y. Chen), `yonghui1676@163.com` (Y. Qin), `huangyq@xtu.edu.cn` (Y. Huang), `hpma@shu.edu.cn` (H. Ma)

1. Introduction

Spectral methods, due to their high-order accuracy, have been developed widely in the numerical solutions of partial differential equations with smooth solutions [7, 15, 31] and the references therein. For problems with variable coefficients, it is well known that the classic spectral method leads to a dense matrix system and it is also not suitable for problems in complex geometries [6]. Recently, the multidomain spectral methods have been applied in many physical models, such as, the supersonic reactive flows [10], the helically reduced wave equations [21], the Schrödinger equations [3], and the references therein. For the problem with discontinuity, the multidomain Legendre Galerkin method has been developed for the one-dimensional evolution equations [24]. There are some reasons to employ multidomain spectral methods for one-dimensional problems as presented in [17].

Some advantages about the least squares finite element method have been given in a review [4]. By introducing a flux, the original problem can be expressed as an equivalent set of first-order equations, and the least-squares finite element methods are developed for partial differential equations in the literatures, e.g., [4, 18]. A symmetric positive definite system can be obtained [4] and it may be solved by a suitable iterative solver, such as the preconditioned conjugate gradient method [4, 30]. Therefore, some least squares finite element methods have been applied in the numerical solutions of the partial differential equations, such as the incompressible resistive magnetohydrodynamics [1] and the Navier-Stokes equations [9].

In [20], Legendre/Chebyshev collocation least-squares methods are developed for elliptic equations. However, as pointed in [20], the error estimate in H^1-norm is not optimal for the Chebyshev spectral method. The Legendre Galerkin coupling with Chebyshev collocation least squares method is developed for the elliptic problem with smooth variable coefficient [28]. Least-squares spectral element method is also applied to the approximation solution of the physics equation, such as the European options [19] and the Stokes equations [25]. A multidomain Legendre-Galerkin Chebyshev collocation least squares method is developed to solve one-dimensional problems with two nonhomogeneous jump conditions in [29]. However, it only considers the error estimate in H^1-norm for β being a piecewise constant coefficients [29].

For simplicity, we first consider a multidomain Legendre-Galerkin least squares methods for solving the following two-point boundary value problems with variable coefficients as

$$\begin{cases} -(\beta p_x)_x + bp_x + cp = f, \quad x \in (0,1), \\ p(0) = p(1) = 0, \end{cases} \tag{1.1}$$

where $\beta(x)$, $b(x)$, and $c(x)$ are smooth functions. For solving Problem (1.1), there are some numerical methods, such as the finite element method [23], the fast structured direct spectral method [32], and the references therein. In this paper, a multidomain Legendre-Galerkin least-squares method is developed for Problem (1.1) with variable coefficients (1.1). An error estimate in H^1-norm can be given, which extends the result