Numer. Math. Theor. Meth. Appl.
doi: 10.4208/nmtma.OA-2020-0072

Vol. **14**, No. 1, pp. 219-241
February 2021

A 2D Staggered Multi-Material ALE Code Using MOF Interface Reconstruction

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Received 13 May 2020; Accepted (in revised version) 3 August 2020

Abstract. Hydrocodes are necessary numerical tools in the fields of implosion and high-velocity impact, which often involve large deformations with changing-topology interfaces. It is very difficult for Lagrangian or Simplified Arbitrary Lagrangian-Eulerian (SALE) codes to tackle these kinds of large-deformation problems, so a staggered Multi-Material ALE (MMALE) code is developed in this paper, which is the explicit time-marching Lagrange plus remap type. We use the Moment Of Fluid (MOF) method to reconstruct the interfaces of multi-material cells and present an adaptive bisection method to search for the global minimum value of the nonlinear objective function. To keep the Lagrangian computations as long as possible, we develop a robust rezoning method named as Combined Rezoning Method (CRM) to generate the convex, smooth grids for the large-deformation domain. Regarding the staggered remap phase, we use two methods to remap the variables of Lagrangian mesh to the rezoned one. One is the first-order intersection-based remapping method that doesn't limit the distances between the rezoned and Lagrangian meshes, so it can be used in the applications of wide scope. The other one is the conservative second-order flux-based remapping method developed by Kucharika and Shashkov [22] that requires the rezoned element to locate in its adjacent old elements. Numerical results of triple point problem show that the result of first-order remapping method using ALE computations is gradually convergent to that of second-order remapping method using Eulerian computations with the decrease of rezoning, thereby telling us that MMALE computations should be performed as few as possible to reduce the errors of the interface reconstruction and the remapping. Numerical results provide a clear evidence of the robustness and the accuracy of this MMALE scheme, and that our MMALE code is powerful for the large-deformation problems.

AMS subject classifications: 65M10, 78A48

Key words: Staggered Lagrangian scheme, multi-material ALE, MOF interface reconstruction, large-deformation problems.

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1. Introduction

Lagrangian hydrodynamic algorithms have been widely used for perhaps the longest time of numerical methods employed for the solution of the complex problem of multi-material fluid flows [8], because they can track the material interfaces clearly and avoid the numerical error of the advection. Moreover, it is easy for Lagrangian schemes to prescribe the boundary conditions, such as free boundary and moving pistons. Lagrangian hydrodynamics algorithms can be classified into the staggered Lagrangian and cell-centered Lagrangian algorithms. Cell-centered Lagrangian algorithms define physical variables and velocities at zone centers [27], which usually need to solve the Riemann problem on the cell boundaries. Staggered Lagrangian algorithms define physical variables of density, internal energy and pressure at zone centers, but velocities are defined at the nodes, which often use the artificial viscosity to stabilize the numerical scheme at the shock [23, 28], or else there will be violent oscillations with the solutions at shocks. Staggered Lagrangian schemes have the overwhelming advantages of being simple and inexpensive [12], so they have been widely used in applications for many years. However, Lagrangian algorithms have the intrinsic difficulty in tackling large-deformation problems with changing-topology interfaces, which can't be tackled by the improved Lagrangian algorithms because they were designed to tackle the unphysical deformations of Lagrangian mesh [8, 11, 12].

Hirt *et al.* [18] put forward to the Arbitrary Lagrangian-Eulerian (ALE) method to integrate the advantages of Lagrangian and Eulerian methods. Prior to the paper [18], the computational fluid dynamicist was limited to Lagrangian and Eulerian methods [3]. Mesh in ALE scheme can move in the styles of Lagrangian, Eulerian or arbitrary providing additional flexibility and accuracy. However, this ALE method in reference [18] was a kind of single-material ALE, which was also called Simplified ALE (SALE) method, in other words, cells could and only could contain one material. Therefore, it could not rezone the mesh across the materials' interfaces, and so couldn't tackle the large-deformation problem with changing-topology interfaces. To overcome the numerical difficulties of large-deformation problems, multi-material ALE method has been developing in recent years, which has been implemented in such production hydrocodes as ALE3D [29], ALEGRA [33] and FLAG [24] etc.

There are two types of ALE methods: direct ALE and indirect ALE. Considering the mesh movements explicitly in the differential equations, direct ALE method has to solve the advection terms of the fluid equations in the form of ALE, the high-order direct ALE scheme can be developed more easily [19], but it is difficult in updating nonlinear physical variables which depend on the history such as stresses and strains. The indirect ALE method is called also as three-stage ALE or split ALE, which is comprised of three stages: (1) Lagrangian stage, in which the solution is updated, and nodes move in Lagrangian style; (2) Rezoning stage, in which the nodes of Lagrangian mesh move to more optimal positions to improve the quality of the mesh; (3) Remapping stage, in which the Lagrangian solution on the Lagrangian mesh is remapped onto the rezoned mesh. Because of the splitting operation, the indirect ALE method is more compatible