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A New Exponential Compact Scheme for the Two-Dimensional Unsteady Nonlinear Burgers' and Navier-Stokes Equations in Polar Cylindrical Coordinates

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Abstract. In this article, a new compact difference scheme is proposed in exponential form to solve two-dimensional unsteady nonlinear Burgers' and Navier-Stokes equations of motion in polar cylindrical coordinates by using half-step discretization. At each time level by using only nine grid points in space, the proposed scheme gives accuracy of order four in space and two in time. The method is directly applicable to the equations having singularities at boundary points. Stability analysis is explained in detail and many benchmark problems like Burgers', Navier-Stokes and Taylor-vortex problems in polar cylindrical coordinates are solved to verify the accuracy and efficiency of the scheme.

AMS subject classifications: 65M06, 65M10, 65Z05, 65Y99

Key words: Half-step discretization, compact scheme in exponential form, two-level implicit scheme, Burgers' equation, Navier-Stokes equations of motion, Taylor-vortex problem.

1. Introduction

Nonlinear parabolic partial differential equations (PDEs) arise in various branches of physical and engineering sciences and play a significant role in computational fluid dynamics (CFD). Two of the most important examples of nonlinear PDEs are the Burg-

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ers' and Navier-Stokes (N-S) equations [3] which have a large variety of applications in studying viscous flow and turbulence and in modeling weather [24]. The Burgers' equation is a special type of nonlinear partial differential equation which is very similar to the N-S equations and could serve as a prototype of the N-S equations. N–S equations are very famous and primordial governing equations in fluid mechanics. Many physical incompressible fluid flow problems are modeled by viscous incompressible N–S equations and numerical computation of these equations is a crucial tool.

During past few decades, tremendous efforts have been made for the development of computational schemes of different numerical approaches to find better approximations of these nonlinear partial differential equations. Finite Difference Method (FDM) [10,14,25] is a very popular numerical approach and many important numerical techniques based on FDM have been developed to solve 2D unsteady nonlinear Burgers' [7] and N-S equations in the polar cylindrical coordinate system. In [2,4,9,12,16] several high order finite difference schemes are proposed to solve 2D unsteady linear parabolic equations numerically depending on alternating direction implicit (ADI) technique. Later, Mohanty and Setia [20] solved the system of quasi-linear parabolic PDEs by using half-step discretization. In order to solve the Navier-Stokes equations for very small Reynolds number, Eren [5] applied the Crank-Nicolson implicit method for the solution of 2D time dependent flow field for interaction of jets. Mohanty *et al.* [15] have developed an operator compact implicit scheme for 1D viscous Burgers-Huxley equation by using three spatial uniform grids and two levels of time. The method has order of accuracy two in time and four in space. Bai *et al.* [1] presented and analyzed approximated ILU and UGS preconditioning methods for the linearized discretized steady incompressible N-S equations. Shah *et al.* [23] proposed a third order upwind compact scheme for the incompressible N-S equations in the artificial compressibility method in general curvilinear coordinates. Finite volume method (FVM) is one of the well-known attractive approaches to tackle with PDEs in complex domain. Pereira *et al.* [22] constructed a fourth order accurate compact scheme based on finite volume method for the incompressible N-S equations in primitive variable formulation. In the field of computational fluid dynamics, high order compact (HOC) formulations are becoming more popular because they provide more accurate solutions in a compact stencil. A class of implicit HOC finite difference schemes have been developed by Kalita *et al.* [11] with weighted time discretization to solve the unsteady 2D variable coefficient convection-diffusion equation. Their methods were tested on problems with both Dirichlet and Neumann boundary conditions. Erturk and Gokcol [6] introduced a new compact formulation of order four in order to solve steady 2D incompressible Navier-Stokes equations. Recently, Mohanty *et al.* [17–19,21] proposed compact HOC method in exponential form in order to solve nonlinear boundary value problems (BVPs).

To our best knowledge no method in exponential form having order accuracy two in time and four in space for the solution of 2D nonlinear parabolic PDEs in cylindrical polar coordinates has been discussed in the literature so far. In this article, we discuss a new half-step discretization in exponential form for the solution of 2D unsteady nonlinear Burgers' and N-S equations in polar cylindrical coordinates. The proposed