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A Hybrid WENO Method with Modified Ghost Fluid Method for Compressible Two-Medium Flow Problems

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Abstract. In this paper, we develop a simplified hybrid weighted essentially non-oscillatory (WENO) method combined with the modified ghost fluid method (MGFM) [31] to simulate the compressible two-medium flow problems. The MGFM can turn the two-medium flow problems into two single-medium cases by defining the ghost fluids state in terms of the predicted the interface state, which makes the material interface "invisible". For the single medium flow case, we adapt between the linear upwind scheme and the WENO scheme automatically by identifying the regions of the extreme points for the reconstruction polynomial as same as the hybrid WENO scheme [55]. Instead of calculating their exact locations, we only need to know the regions of the extreme points based on the zero point existence theorem, which is simpler for implementation and saves computation time. Meanwhile, it still keeps the robustness and has high efficiency. Extensive numerical results for both one and two dimensional two-medium flow problems are performed to demonstrate the good performances of the proposed method.

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Key words: Hybrid WENO scheme, two-medium flow problems, modified ghost fluid method, zero point existence theorem.

1. Introduction

In this paper, we propose a simplified hybrid weighted essentially non-oscillatory

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(WENO) method with modified ghost fluid method (MGFM) [31] for simulating compressible two-medium flow problems. For two-medium flow problems, the equation of state (EOS) would switch between the different medium, which may cause numerical oscillations or inaccuracies near the material interface. Hence, many researchers have used various additional works and techniques to overcome this difficulty, and there are two major options to simulate the compressible two-medium flow problems.

One is the front capturing method, where the high resolution methods are applied to suppress the non-physical oscillations near discontinuities by bringing the numerical diffusion or viscosity, which inherently exists in the method itself or is given artificially. The front capturing method can deal with large deformation problems and relatively easy to extend to high dimension. However, the numerical inaccuracies and oscillations are inevitable near the interface, therefore, various techniques were introduced by Larrouturou [22], Karni [21], Abgrall [1], Abgrall and Karni [2], Shyue [44], Saurel and Abgrall [39], Chen and Jiang [5] to resolve this difficulty. The other one is the front tracking method, which terms the discontinuities between the two-medium flows as internal moving interfaces. It works well at multi-material interfaces, but it would have difficulties about the entanglement of the Lagrangian meshes and the extension to high dimension, and there are some typical methods about the front tracking method, such as volume of fluid (VOF) method [13], level set method [46] and other front tracking methods [7, 47].

To combine the best properties of the front capturing and tracking methods, Fedkiw *et al.* [6] proposed a new numerical method for treating interfaces using a level set function in Eulerian schemes named as the ghost fluid method (GFM), which makes the interface "invisible". In the framework of the GFM [6], the pressure and velocity at the ghost fluid nodes near the interface are defined as the local real pressure and velocity, while the density is obtained by isobaric fixing. It can easily turn the two-medium flow problems into two single-medium flow cases, and for the single-medium flow problems, many classical and mature schemes can be applied. Hence, it provides an alternative and flexible way to simulate the two-medium flow problems, and the extension to high dimension becomes fairly straightforward. However, it may cause numerical inaccuracies in the case of a strong shock impacting on the interface, and the reason may be that the states near the interface are affected by the wave interaction and the material properties on both sides. Therefore, Liu *et al.* [31] developed a modified ghost fluid method (MGFM), in which a multi-material Riemann problem is defined and solved approximately or exactly to predict the interfacial state, then, the predicted interfacial state is applied to define the fluid values at the ghost points. The MGFM combines the advantages of the GFM [6] and the implicit characteristic methods [29, 30], and it takes the interaction of shock with the interface into consideration. Later, the interface interaction GFM (IGFM) [15], the real GFM (RGFM) [48] and the practical GFM (PGFM) [49] were developed following the idea of the Riemann problem-based technique in the MGFM [31]. The MGFM is robust and less problem related, and it has been applied in various situations as in [26, 28, 32, 36, 51], and the accuracy analysis and errors estimation can be seen in [27, 50]. The GFM [6] and its relevant ghost fluid