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Phaseless Imaging by Reverse Time Migration: Acoustic Waves

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Abstract. We propose a reliable direct imaging method based on the reverse time migration for finding extended obstacles with phaseless total field data. We prove that the imaging resolution of the method is essentially the same as the imaging results using the scattering data with full phase information when the measurement is far away from the obstacle. The imaginary part of the cross-correlation imaging functional always peaks on the boundary of the obstacle. Numerical experiments are included to illustrate the powerful imaging quality.

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1. Introduction

We consider in this paper inverse scattering problems with phaseless data which aim to find the support of unknown obstacles from the knowledge of the amplitude of the total field measured on a given surface far away from the obstacles. Let the sound soft obstacle occupy a bounded Lipschitz domain $D\subset\mathbb{R}^2$ with ν the unit outer normal to its boundary Γ_D. Let u^i be the incident wave and the total field is $u=u^i+u^s$ with u^s being the solution of the following acoustic scattering problem:

$$\Delta u^s+k^2u^s=0 \quad \text{in } \mathbb{R}^2\backslash\bar{D}, \tag{1.1}$$

$$u^s=-u^i \quad \text{on } \Gamma_D, \tag{1.2}$$

$$\sqrt{r}\Big(\frac{\partial u^s}{\partial r}-\mathbf{i}ku^s\Big)\to 0 \text{ as } r=|x|\to+\infty, \tag{1.3}$$

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where $k > 0$ is the wave number. The condition (1.3) is the outgoing Sommerfeld radiation condition which guarentees the uniqueness of the solution. In this paper, by the radiation or scattering solution we always mean the solution satisfies the Sommerfeld radiation condition (1.3). For the sake of the simplicity, we mainly consider the imaging of sound soft obstacles in this paper. The extension of our theoretical results for imaging other types of obstacles will be briefly considered in Section 4.

In the diffractive optics imaging and radar imaging systems, it is much easier to measure the intensity of the total field than the phase information of the field [11, 12, 36]. It is thus very desirable to develop reliable numerical methods for reconstructing obstacles with only phaseless data, that is, the amplitude information of the total field $|u|$. In recent years, there have been considerable efforts in the literature to solve the inverse scattering problems with phaseless data. One approach is to image the object with the phaseless data directly in the inversion algorithm, see e.g. [12, 24, 25]. The other approach is first to apply the phase retrieval algorithm to extract the phase information of the scattering field from the measurement of the intensity and then use the retrieved full field data in the classical imaging algorithms, see e.g. [13]. In [19, 20], explicit formulas are provided for the reconstruction by using Radon transform. Theoretical analysis of the phaseless inverse scattering problems and reconstruction procedures with the scattering amplitude are provided in [29, 30]. Resolution and stability analysis together with reconstruction procedures by phaseless data are obtained in [1] by using the concept of scattering coefficients based on the linearization method. We also refer to [2] for the continuation method and [16, 17, 22] for inverse scattering problems with the data of the amplitude of the far field pattern. In [18] some uniqueness results for phaseless inverse scattering problems have been obtained.

The reverse time migration (RTM) method, which consists of back-propagating the complex conjugated scattering field into the background medium and computing the cross-correlation between the incident wave field and the backpropagated field to output the final imaging profile, is nowadays a standard imaging technique widely used in seismic imaging [3]. Let $\Gamma_s = \partial B_s$ and $\Gamma_r = \partial B_r$, where B_s, B_r are the disks of radius R_s, R_r respectively. Without loss of generality, we assume $R_r \geq R_s$. We denote by Ω the sampling domain in which the obstacle is sought. Let $u^i(x, x_s) = \Phi(x, x_s)$, where $\Phi(x, x_s) = \frac{\mathbf{i}}{4} H_0^{(1)}(k|x - x_s|)$ is the fundamental solution of the Helmholtz equation with the source at $x_s \in \Gamma_s$, be the incident wave and $u^s(x, x_s)$ is the solution to the problem (1.1)-(1.3) with $u^i(x, x_s) = \Phi(x, x_s)$. The RTM imaging function studied in [4] for reconstructing extended targets is

$$I_{\mathrm{RTM}}(z) = -k^2 \mathrm{Im} \int_{\Gamma_s} \int_{\Gamma_r} \Phi(z, x_s) \Phi(x_r, z) \overline{u^s(x_r, x_s)} ds(x_r) ds(x_s), \quad \forall\, z \in \Omega, \tag{1.4}$$

where Ω is the sampling domain that the obstacle is sought. It is shown in [4] that this imaging function always peaks on the boundary of the obstacle. In [5, 6], the RTM methods for reconstructing extended targets using electromagnetic and elastic waves at a fixed frequency are proposed and studied.