Numer. Math. Theor. Meth. Appl.
doi: 10.4208/nmtma.2017.s05

Vol. **10**, No. 2, pp. 278-298
May 2017

Multidimensional Iterative Filtering Method for the Decomposition of High–Dimensional Non–Stationary Signals

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Received 7 November 2016; Accepted (in revised version) 26 January 2017

Dedicated to Professor Zhenhuan Teng on the occasion of his 80th birthday

Abstract. Iterative Filtering (IF) is an alternative technique to the Empirical Mode Decomposition (EMD) algorithm for the decomposition of non–stationary and non–linear signals. Recently in [3] IF has been proved to be convergent for any L^2 signal and its stability has been also demonstrated through examples. Furthermore in [3] the so called Fokker–Planck (FP) filters have been introduced. They are smooth at every point and have compact supports. Based on those results, in this paper we introduce the Multidimensional Iterative Filtering (MIF) technique for the decomposition and time–frequency analysis of non–stationary high–dimensional signals. We present the extension of FP filters to higher dimensions. We prove convergence results under general sufficient conditions on the filter shape. Finally we illustrate the promising performance of MIF algorithm, equipped with high–dimensional FP filters, when applied to the decomposition of two dimensional signals.

AMS subject classifications: 60G35, 65D05, 65T60,62H35

Key words: Iterative filtering, multidimensional iterative filtering, empirical mode decomposition, non–linear and non–stationary multidimensional signals, fokker–planck filters.

1. Introduction

Given a non–stationary signal a hard problem is how to perform a time–frequency analysis in order to unravel its hidden features. The problem becomes even harder if we want to handle a signal that has dimension higher than one. Such kind of problems are ubiquitous in real life and their solutions can help shedding lights in many research fields. We mention, for instance, the non–destructive detection of structural damages

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in buildings and machineries, the quantitative canvas weave analysis in the art investigations of paintings, the identification of harmful airborne chemical particles by means of hyperspectral images analysis, the image enhancement for medical applications, the extraction of information from atomic crystal images, et cetera.

Time–frequency analysis has been substantially studied in the past [6] and, traditionally, linear techniques like Fourier spectral analysis or wavelet transforms have been commonly used to decompose signals, even non–stationary ones, into simple and stationary components and then perform on them a frequency analysis. These approaches, even if easy to implement, have limitations. In particular, they work well when the given signal is periodic and stationary, whereas they cannot handle properly non–stationary signals. Furthermore all these techniques use predetermined bases and they are not designed to be data–adaptive. Hence there is the need to develop new methods designed to handle specifically non–linear and non–stationary signals.

The research on these new techniques started in 1998 when Huang and his research group developed and released the very first method of this kind, the so called Empirical Mode Decomposition (EMD) algorithm [14]. The idea behind this method is to generalize the approach of classical linear techniques: decompose a signal into simple components and then perform a time–frequency analysis on each component separately. For this reason Huang and his group defined the so called Intrinsic Mode Functions (IMFs) which are simple functions with the following properties: their number of extrema is either equal to the number of zero crossings or they differ at most by one, and at any point their moving average is zero. Furthermore they developed an iterative technique, called the sifting process, to decompose a signal into such Intrinsic Mode Functions with the final goal of computing the instantaneous frequency [2, 6] of these simple components.

The sifting process of the EMD is structured in the following way. Let L be an operator capturing the moving average of a signal f, and let

$$f_2 = S_{1,1}(f)(t) = f(t) - L(f)(t)$$

be its fluctuation part. If we iterate this procedure to compute

$$f_n = S_{1,n-1}(f_{n-1})(t) = f_{n-1}(t) - L(f_{n-1})(t),$$

then the first IMF is given by $\mathrm{IMF}_1(t) = \lim_{n\to\infty} S_{1,n}(f)(t)$, where the subscripts 1 and n stand for the first IMF and n–th iteration. The limit is achieved when the moving average of $\mathrm{IMF}_1(t)$ is the zero function. Assuming $k \geq 1$ IMFs have been computed, then the subsequent IMFs are produced applying the aforementioned procedure to the residual

$$r(t) = f(t) - \sum_{j=1}^{k} \mathrm{IMF}_j(t).$$

The method stops when the residual r becomes a trend signal. So in the end the given