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A Few Benchmark Test Cases for Higher-Order Euler Solvers

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Abstract. There have been great efforts on the development of higher-order numerical schemes for compressible Euler equations in recent decades. The traditional test cases proposed thirty years ago mostly target on the strong shock interactions, which may not be adequate enough for evaluating the performance of current higher-order schemes. In order to set up a higher standard for the development of new algorithms, in this paper we present a few benchmark cases with severe and complicated wave structures and interactions, which can be used to clearly distinguish different kinds of higher-order schemes. All tests are selected so that the numerical settings are very simple and any higher order scheme can be straightforwardly applied to these cases. The examples include highly oscillatory solutions and the large density ratio problem in one dimensional case. In two dimensions, the cases include hurricane-like solutions; interactions of planar contact discontinuities with asymptotic large Mach number (the composite of entropy wave and vortex sheets); interaction of planar rarefaction waves with transition from continuous flows to the presence of shocks; and other types of interactions of two-dimensional planar waves. To get good performance on all these cases may push algorithm developer to seek for new methodology in the design of higher-order schemes, and improve the robustness and accuracy of higher-order schemes to a new level of standard. In order to give reference solutions, the fourth-order gas-kinetic scheme (GKS) will be used to all these benchmark cases, even though the GKS solutions may not be very accurate in some cases. The main purpose of this paper is to recommend other CFD researchers to try these cases as well, and promote further development of higher-order schemes.

AMS subject classifications: 65M10, 78A48

Key words: Euler equations, two-dimensional Riemann problems, fourth-order gas-kinetic scheme, wave interactions.

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1. Introduction

In past decades, there have been tremendous efforts on designing high-order accurate numerical schemes for compressible flows and great success has been achieved. The developments of high-order accurate numerical schemes were pioneered by Lax and Wendroff [22], and extended into the version of high resolution methods by Kolgan [18], Boris [7], van Leer [44], Harten [13] et al, and other higher order versions, such as Essentially Non-Oscillatory (ENO) [16, 41], Weighted Essentially Non-Oscillatory (WENO) [17, 31], Discontinuous Galerkin (DG) [10, 12, 35] methods etc.

In the past decades, the evaluation of the performance of numerical scheme was mostly based on the test cases with strong shocks for capturing sharp shock transition, such as the blast wave interaction, the forward step-facing flows, and the double Mach reflection [45]. Now it is not a problem at all for shock capturing scheme to get stable sharp shock transition. However, with the further development of higher order numerical methods and practical demands (such as turbulent flow simulations), more challenging test problems for capturing multiple wave structure are expected to be used. For testing higher-order schemes, the setting of these cases should be sufficiently simple and easy for coding, and avoid the possible pollution from the boundary condition and curvilinear meshes. To introduce a few tests which can be truthfully used to evaluate the performance of higher-order scheme is the motivation of the current paper. Our selected examples include the following ones: one-dimensional cases, two-dimensional Riemann problems, and the conservation law with source terms. For the one-dimensional problems, the first case is a highly oscillatory shock-turbulence interaction problem, which is an extension of Shu-Osher problem by Titarev and Toro [43] with much more severe oscillations, and the second one is a large density ratio problem with a very strong rarefaction wave in the solution [42], which can be used to test the robustness and accuracy in capturing strong expansion waves. For the two-dimensional cases, four group wave interactions are tested. (i) Hurricane-like solutions [27, 48], which are highly nontrivial two-dimensional time-dependent solutions with one-point vacuum in the center and rotational velocity field. It is proposed to test the preservation of positivity and symmetry of the numerical scheme. (ii) The interaction of planar contact discontinuities for different Mach numbers. The multidimensional contact discontinuities are the composite of entropy waves and vortex sheets. The simulation of these cases is associated with difficulties for capturing the strong shear effects. Since at the large Mach number limits these cases have explicit solutions [27, 40], they are proposed here to check the ability of higher-order schemes in capturing wave structures of various scales and the asymptotic property. (iii) Interaction of planar rarefaction waves with the transition from continuous fluid flows to the presence of shocks. (iv) Further interaction of planar shocks with Mach reflection phenomenon. These two-dimensional problems fall into the category of two-dimensional Riemann problems proposed in [49]. The two-dimensional Riemann problems reveal almost all substantial wave patterns of shock reflections, spiral formations, vortex-shock interactions, and so on, through simple classification of initial data. The rich wave configurations conjectured in [49] have been confirmed numerically by several subsequent works [14, 20, 27, 37]. Since the formula-