Numer. Math. Theor. Meth. Appl. Vol. **11**, No. 2, pp. 299-320
doi: 10.4208/nmtma.OA-2017-0090 May 2018

A Linearized High-Order Combined Compact Difference Scheme for Multi-Dimensional Coupled Burgers' Equations

Buyun Chen[1], Dongdong He[2,*] and Kejia Pan[3]

[1] *School of Aerospace Engineering and Applied Mechanics, Tongji University, Shanghai 200092, China*
[2] *School of Science and Engineering, the Chinese University of Hong Kong, Shenzhen 518172, China*
[3] *School of Mathematics and Statistics, Central South University, Changsha 410083, China*

Received 22 July 2017; Accepted (in revised version) 9 November 2017

Abstract. Multi-dimensional coupled Burgers' equations are important nonlinear partial differential equations arising in fluid mechanics. Developing high-order and efficient numerical methods for solving the Burgers' equation is essential in many real applications since exact solutions can not be obtained generally. In this paper, we seek a high-order accurate and efficient numerical method for solving multi-dimensional coupled Burgers' equations. A linearized combined compact difference (CCD) method together with alternating direction implicit (ADI) method is proposed. The CCD-ADI method is sixth-order accuracy in space variable and second-order accuracy in time variable. The resulting linear system at each ADI computation step corresponds to a block-tridiagonal system which can be effectively solved by the block Thomas algorithm. Fourier analysis shows that the method is unconditionally stable. Numerical experiments including both two-dimensional and three-dimensional problems are conducted to demonstrate the accuracy and efficiency of the method.

AMS subject classifications: 76M20, 65M06

Key words: Combined compact difference scheme, coupled Burgers' equations, linearized method, ADI method, unconditional stability.

1. Introduction

It is well known that Burgers' equation is an important partial differential equation in fluid mechanics, which has been used for a variety of applications, such as traffic flow [1], gas dynamics [2], shock waves [3], wave propagation in acoustics [4], shallow water waves, etc. In particular, one-dimensional (1D) Burgers' equation is used to model the

*Corresponding author. *Email addresses:* 1531892@tongji.edu.cn (B. Chen), hedongdong@cuhk.edu.cn (D. He), pankejia@hotmail.com (K. Pan)

water in unsaturated oil, dynamics of soil in water, cosmology and seismology [5–7], and 1D coupled viscous Burgers' equation was derived by Esipov [8] to study the model of poly-dispersive sedimentation. Two-dimensional (2D) coupled Burgers' equations are used in the natural applications such as modeling of gas dynamics [2] and investigating the shallow water waves [3, 9]. Three-dimensional (3D) coupled Burgers' equations are used as an adhesive model for the large scale structure formation in the universe [10]. Under a special set of initial and boundary conditions, the nonlinear Burgers' equation can be solved analytically. The exact solution for 1D Burgers' equation was first obtained by Bateman [11]. Hopf [12] and Cole [13] independently showed that 1D Burgers' equation can be transformed to a linear heat equation such that it can be solved exactly under the homogeneous boundary conditions. The 1D Hopf-Cole transformation was recently generalized to solve the Painleve III equation and the 1D generalized Burgers' equation by Humi [14]. When the initial vorticity is zero, by using 2D version of Hopf-Cole transformation, Fletcher [15] generated the exact solution for 2D coupled Burgers' equations. More recently, the 3D version of Hopf-Cole transformation was also used [16, 17] for the 3D coupled Burgers' equations to derive exact solutions under the condition that the initial vorticity is zero. However, for the general initial boundary conditions, Burgers' equation can not be solved exactly due to its nonlinear nature. Thus, numerical computations become essentially important.

In the existing literature, great efforts have been devoted to solve the Burgers' equation and its variational form by using different numerical methods [18–33]. The 1D Burgers' equation and its variational form were numerically extensively studied in the literature [18–27]. For example, Chan and Chung [18] used the staggered discontinuous Galerkin method for solving the 1D inviscid Burgers' equation. Dehghan [19] solved the 1D viscous Burgers' equation and 1D coupled viscous Burgers' equations by using the mixed finite difference and Galerkin methods. Kutluay and Ucar [20] solved the 1D coupled viscous Burgers' equations by the Galerkin quadratic B-spline finite element method. Varöglu [21] and Caldwell et al. [22] used finite element methods for solving the 1D viscous Burgers' equation. Bahadir [23] utilized a mixed finite difference and boundary element approach for solving the 1D viscous Burgers' equation. Sari and Görarslan [24] applied a sixth-order compact finite difference scheme for spatial discretization and a Runge-Kutta scheme explicit method for time integration to solve the 1D viscous Burgers' equation. Liao [26, 27] applied the Hopf-Cole transformation to 1D viscous Burgers' equation first and then solved the resulting linear heat equation by using the standard fourth-order three-point compact finite difference method. For 2D coupled Burgers' equations, there are also quite a lot of existing studies [28–31]. For example, Huang and Abduwali [28] proposed a modified local Crank-Nicolson method, where it is second-order accurate in space variable and first-order accurate in time variable. Radwan [29] solved the 2D viscous Burgers' equations by using the fourth-order accurate two-point compact alternating direction implicit (ADI) scheme and the fourth-order Du Fort Frankel scheme. Bahadir [30] used a fully implicit finite difference scheme for solving the 2D viscous Burgers' equations, and some iteration methods are needed in order to get the convergent numerical results at each time step. Moreover, Liao [31] applied 2D Hopf-Cole transformation to 2D coupled viscous