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The Modulus-Based Levenberg-Marquardt Method for Solving Linear Complementarity Problem

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Abstract. As applying the Levenberg-Marquardt method to the reformulation of linear complementarity problem, a modulus-based Levenberg-Marquardt method with non-monotone line search is established and the global convergence result is presented. Numerical experiments show that the proposed method is efficient and outperforms the modulus-based matrix splitting iteration method.

AMS subject classifications: 90C33, 65F10

Key words: Linear complementarity problem, modulus-based, Levenberg-Marquardt method, global convergence, numerical experiments.

1. Introduction

Let $\mathbb{R}^n$ and $\mathbb{R}^{n\times n}$ be the n-dimensional real vector space and the n-by-n real matrix space, respectively. In this paper, we consider the linear complementarity problem, abbreviated as LCP(q,M), for finding a pair of real vectors w and $z \in \mathbb{R}^n$ such that

$$w := Mz + q \geq 0, \quad z \geq 0 \quad \text{and} \quad z^T w = 0, \tag{1.1}$$

where $M = (m_{ij}) \in \mathbb{R}^{n\times n}$ is a given large, sparse and real matrix, and

$$q = (q_1, q_2, \cdots, q_n)^T \in \mathbb{R}^n$$

is a given real vector. Here, the notation $\geq$ denotes the componentwise defined partial ordering between two vectors and the superscript T denotes the transpose of a vector.

The linear complementarity problem was introduced by Lemke in 1964, but it was Cottle and Dantzig [1] who formally defined the linear complementarity problem and called it the fundamental problem. The LCP(q,M) of the form (1.1) often arises in many scientific computing and engineering applications, e.g., the linear and quadratic programming, the economies with institutional restrictions upon prices, the optimal stopping in Markov chain, and the free boundary problems; see [2,3,5] for details.

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For the solution of the large and sparse LCP(q, M), the pivot algorithms based on simplex type processes require too many pivots, destroy sparsity, have exponential computational complexity and suffer from round-off errors [10]. Therefore, iterative methods, such as projected relaxation method [11], were constructed and widely discussed. Mangasarian [12] and Ahn [13] established the convergence theory of the projected iterative method when the matrix is either symmetric or nonsymmetric.

By equivalently reformulating the LCP(q, M) as an implicit fixed-point equation, Van Bokhoven [6] presented a modulus iteration method, which was defined as the solution of linear equations at each iteration. Moreover, Bai [7] presented a class of modulus-based matrix splitting iteration methods which not only provided a general framework for the modified modulus method [8] and nonstationary extrapolated modulus algorithms [9], but also yielded a series of modulus-based relaxation methods which outperform the projected relaxation method as well as the modified modulus method in computing efficiency. With respect to matrix splitting method and modulus-based method, we can also refer to [18, 20–23, 25, 26, 28–33] and the references therein.

As we all know, the implicit fixed-point equation which is equivalent to the LCP(q, M) is a absolute value equation. Iqbel et al. [14] proposed Levenberg-Marquardt method for solving absolute value equations, which is the combination of steepest descent and the Gauss-Newton methods. They proved the global convergence of new method when using the Goldstein line search. Li and Fukushima [15] presented a non-monotone line search for nonlinear equations, that is

$$\left\|F(x_k + \alpha d_k)\right\|^2 \leq (1 + \eta_k)\|F(x_k)\|^2 - \sigma_1 \alpha^2 \|d_k\|^2 - \sigma_2 \alpha^2 \|F(x_k)\|^2, \tag{1.2}$$

where $F(x) : \mathbb{R}^n \to \mathbb{R}^n$ is a continuous function, σ_1 and σ_2 are positive constants and the positive sequence $\{\eta_k\}$ satisfies

$$\sum_{k=0}^{\infty} \eta_k < \infty. \tag{1.3}$$

It is noticeable that as $\alpha \to 0^+$, the left hand side of (1.2) goes to $\|F(x_k)\|^2$, while the right hand side tends to the positive constant $(1 + \eta_k)\|F(x_k)\|^2$. Thus, (1.2) is satisfied for all sufficiently small $\alpha > 0$. Hence, one can obtain α_k by means of a backtracking process. This non-monotone line search can guarantee the global convergence of the Levenberg-Marquardt method [19].

Inspired by the above mentioned, we present the Levenberg-Marquardt method with a non-monotone line search for the LCP(q, M).

The outline of this paper is as follows. We give some basic notations, definitions and lemmas in Section 2 and establish the modulus-based Levenberg-Marquardt method for linear complementarity problem in Section 3. In Section 4, the global convergence of the modulus-based Levenberg-Marquardt method is proved. In Section 5, the numerical experiments are presented to show the effectiveness of our method. In the final section we give the concluding remarks.