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Numerical Investigations of a Class of Biological Models on Unbounded Domain

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Abstract. This paper is concerned with numerical computations of a class of biological models on unbounded spatial domains. To overcome the unboundedness of spatial domain, we first construct efficient local absorbing boundary conditions (LABCs) to reformulate the Cauchy problem into an initial-boundary value (IBV) problem. After that, we construct a linearized finite difference scheme for the reduced IVB problem, and provide the corresponding error estimates and stability analysis. The delay-dependent dynamical properties on the Nicholson's blowflies equation and the Mackey-Glass equation are numerically investigated. Finally, numerical examples are given to demonstrate the efficiency of our LABCs and theoretical results of the numerical scheme.

AMS subject classifications: 34A45, 65M12, 65N12

Key words: Biological models, artificial boundary method, absorbing boundary conditions, finite difference method, stability, convergence.

1. Introduction

We consider the numerical computation of model equations with delay term on the unbounded spatial domain given as

$$u_t = u_{xx} - du + f(u(x,t-\tau)), \quad x \in \mathbb{R}, \;\; t \in (0,T], \tag{1.1}$$

where the initial value is supposed to be compactly supported

$$u(x,t) = u_0(x,t), \quad (x,t) \in \mathbb{R} \times [-\tau, 0].$$

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Model (1.1) was first proposed to demonstrate the distribution of the Australian blowflies, where $u(x,t)$ represents the mature population of the blowflies, $d > 0$ denotes the death rate of the mature population, the delay term τ represents the time required for a newborn to become matured, the diffusion is used to describe the spatial movement of substances from high to low concentration, and $f(u)$ is the birth-rate function. Two typical birth-rate functions are given as

$$f(u) = pue^{-au}, \tag{1.2a}$$

$$f(u) = \frac{pu}{1+u^2}, \tag{1.2b}$$

where p is the impact of the birth on the immature population. Model (1.1) with the birth-rate function (1.2a) is usually called Nicholson's blowflies equation, and with the function (1.2b) called the Mackey-Glass equation. It is remarkable that this kind of delay equations (1.1) are also considered as models in many other applied scientific fields. One can refer to [1,3,31] for details.

PDEs with delay have been extensively studied. Most results in the literature indicate that a delay term has an important impact on the dynamic properties of a system such as stability, dissipativity, chaos etc. For example, Lin et al. [19] showed that the traveling wave changes with the delay term by using the weighted energy method. Huang and Vandewalle [9] studied the delay-dependent stability of continuous and discrete systems. Strogatz [25] considered the nonlinear dynamic behaviors which is related to the delay term. Zou et al. [45] studied the relation between the delay term and oscillatory behaviors in diffusively coupled dynamical networks. Generally, the investigation on delay-dependent dynamic properties of a system is believed to be one of the most challengeable work. One important reason is that the exact solutions of these PDEs with delay are difficult to obtain.

For the bounded domain case, there are many studies on the numerical simulation of linear and nonlinear PDEs with delay. Jackiewicz and Zubik-Kowal [11] investigated Chebyshev spectral collocation and waveform relaxation methods. Sun [27] applied the linearized compact difference scheme. Zhang and Zhang [38] solved the parabolic problems with delay by combining the compact finite difference method with different time discretization (see also [15,18,37,39]). Li and Zhang [16,17] introduced the discontinuous Galerkin methods. Zhang and Xiao [37] proposed the implicit-explicit finite difference methods.

For the unbounded domain case, although the theory of problem (1.1) has been well studied (see [5,19,23]), the numerical analysis for the problem (1.1) has so far received little attention because the unboundedness of the definition domain usually presents a great numerical difficulty. To deal with the unboundedness, one of the powerful tools is to employ artificial boundary methods (ABMs), see the monograph by [8]. The main idea of ABMs is to limit a bounded computational domain of interest by introducing artificial boundary, then impose suitable absorbing boundary conditions (ABCs) on the artificial boundaries, and finally reformulate the unbounded problem into a bounded problem. As repeatedly shown by different authors both theoretically and experimentally, the overall accuracy and performance of numerical schemes strongly depend on the choice of ABCs