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The Instability in the Dimensions of Spline Spaces Over T-Meshes with Nested T-Cycles

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Abstract. This paper studies on the dimensions of spline spaces over some given T-meshes. Using the smoothing cofactor-conformality method, we study the instability in the dimensions of the spline spaces over T-meshes with 2-nested and 3-nested T-cycles. We define a singularity factor of each simple T-cycle, the instability and the structure's degeneration are associated with the singularity factors. In order to get a stable dimension formula over T-mesh with a N-nested T-cycle, a constraint on the T-mesh is introduced. Finally, a possible degeneration for a case of parallel T-cycles is illustrated.

AMS subject classifications: 65D07
Key words: Spline space, smoothing cofactor-conformality method, instability in the dimension, T-mesh, T-cycle.

1. Introduction

As an industry standard, the Non-Uniform Rational B-spline (NURBS) is commonly used in computer-aided design (CAD), manufacturing (CAM), and engineering (CAE). However, NURBS models suffer from a major weakness that they contain a large number of superfluous control points [1, 2]. To overcome this limitation, the T-spline, a point-based spline defined on T-mesh, was invented by Sederberg et al. [1]. It facilitates local refinement and makes the representation of free-form surfaces more flexible. In addition, a class of T-splines called Analysis-suitable T-splines has been proposed, for which basis functions are linearly independent and form a partition of unity [3,4]. Scott et al. [4] also gave a greedy algorithm to achieve the local refinement for Analysis-suitable T-splines.

Besides, there is another way to obtain local modification on the tensor-product B-splines over rectangular meshes. The spline space over a T-mesh, denoted by $S(m,n,\alpha,\beta,\mathscr{T})$, was firstly introduced in [5], which is a bi-degree (m,n) piecewise polynomial spline space over T-mesh $\mathscr{T}$ with smoothness order α along x direction and β in y direction. By using the B-net method, Deng et al. [5] calculated the dimension of spline

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space $S(m, n, \alpha, \beta, \mathscr{T})$ with constraints of $m \geq 2\alpha + 1$ and $n \geq 2\beta + 1$ when the T-mesh has no T-cycle. Huang et al. [6] derived an equivalent dimension formula in a different form by the smoothing cofactor-conformality method [7,8]. Li et al. [9,10] improved the dimension formula in the same spline space by using the smoothing cofactor-conformality method with a constraint depending on the order of the smoothness, the degree of the spline functions and the structure of the T-mesh as well. Further, Li et al. [11] discussed the stability of the dimensions of general spline spaces based on the analysis of the conformality condition at one interior vertex. Mourrain [12] defined the weighted T-meshes, over which the dimension could be computed in an explicit formula by using the homological techniques. Li et al. [13] introduced a class of T-meshes, diagonalizable T-meshes, over which the dimensions of spline spaces are stable. Wang et al. [14] proposed a de Boor like algorithm to evaluate PHT-splines–polynomial splines over hierarchical T-meshes with stable dimensions. However, the case of instable dimension of the spline space over T-mesh was discussed insufficiently.

As we known, for the Morgan-Scott triangulation, the dimension of spline space depends on not only the topological information of the partition but also the geometry structure of the partition [15]. Recently, Li et al. [16] and Berdinsky et al. [17] found the instability of the dimensions of spline spaces over some certain T-meshes. In 2015, Guo et al. [18] discovered the instability of the dimension for the T-mesh with an independent simple T-cycle. However, if several T-cycles are connected to each other or to say dependent to each other, it is difficult to determine the associated dimension. In this paper, we study on the dimensions of spline spaces over some given T-meshes with nested T-cycles and parallel T-cycles which are composed of several connected T-cycles. We find some very interesting results on the dimensions of the corresponding spline spaces. For the cases of 2-nested and 3-nested T-cycles, the instability is associated with corresponding singularity factor of each T-cycle. In addition, we give a stable dimension formula over T-mesh with a N-nested T-cycle by a constraint on the T-mesh.

The remainder of this paper is organized as follows. We review on some definitions and notations regarding the T-meshes and give the definition of N-nested T-cycle in Section 2. In Section 3, we discuss the instability on the dimensions of spline spaces over T-meshes with 2-nested and 3-nested T-cycles respectively. Moreover, we show that the instability of the dimension and the degeneration of the corresponding structure of the T-mesh are related to a geometric condition which is defined as singularity factor and we introduce a constraint on the T-mesh in order to obtain a stable dimension formula for T-mesh with a N-nested T-cycle. In Section 4, a case of parallel T-cycles on the degeneration of T-mesh is presented. Finally, conclusions are drawn in the last section.

2. Preliminaries

In this section, we briefly review some definitions and notations of T-mesh as in [9], then give some prior work on the instability in the dimension of spline space.

A T-mesh is a modified rectangular grid that allows T-junctions arising from T-spline. For a given T-mesh as shown in Fig. 1(a), its interior mesh segments are divided into three