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Modulus-Based Synchronous Multisplitting Iteration Methods for a Restricted Class of Nonlinear Complementarity Problems

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Abstract. A class of nonlinear complementarity problems are first reformulated into a series of equivalent implicit fixed-point equations in this paper. Then we establish a modulus-based synchronous multisplitting iteration method based on the fixed-point equation. Moreover, several kinds of special choices of the iteration methods including multisplitting relaxation methods such as extrapolated Jacobi, Gauss-Seidel, successive overrelaxation (SOR), and accelerated overrelaxation (AOR) of the modulus type are presented. Convergence theorems for these iteration methods are proven when the coefficient matrix A is an H_+-matrix. Numerical results are also provided to confirm the efficiency of these methods in actual implementations.

AMS subject classifications: 65M10, 78A48

Key words: Nonlinear complementarity problem, modulus-based synchronous multisplitting, iteration method, H_+-matrix, H-compatible splitting, convergence.

1. Introduction

There are a large number of problems in scientific computing and application engineering demanding to compute solutions of complementarity problems. For example, some of those are found in contact problem in elasticity, economic transportation, and the free boundary problem of fluid dynamics, see, e.g., [1, 2].

Let $\mathbb{R}^n$ and $\mathbb{R}^{n\times n}$ be n-dimensional real vector space and real matrix space, respectively. Our paper will focus on considering the following nonlinear complementarity problem denoting by ($NCP(q,A)$) [3, 4], which aims to find a pair of real vectors $u, v \in \mathbb{R}^n$ such that

$$u \geq 0, \quad v := Au + q + \Phi(u) \geq 0, \quad u^T v = 0, \tag{1.1}$$

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where $A = (a_{ij}) \in \mathbb{R}^{n\times n}$ and $q = (q_1, q_2, \cdots, q_n) \in \mathbb{R}^n$ are given sparse real matrix and real vector, respectively. Assume that the $\Phi : \mathbb{R}^n \to \mathbb{R}^n$ is a given diagonal differentiable mapping with nonnegative partial derivatives, that is, each of the i th component Φ_i of Φ is a function of the i th variable u_i: $\Phi_i = \Phi_i(u_i)$, and $\frac{\partial\Phi_i(u_i)}{\partial u_i} \geq 0, 1 \leq i \leq n$. Meanwhile, there exists a positive diagonal matrix $\widetilde{D}_n$ such that

$$D_n(u) := diag\left(\frac{\partial\Phi_1(u_1)}{\partial u_1}, \frac{\partial\Phi_2(u_2)}{\partial u_2}, \cdots, \frac{\partial\Phi_n(u_n)}{\partial u_n}\right) \leq \widetilde{D}_n. \tag{1.2}$$

Moreover, the notation '$\geq$' denotes the componentwise partial ordering of vectors as is explained in the beginning of Section 2; and the superscript 'T' denotes the transpose of a vector.

Obviously, problem (1.1) reduces to a linear complementarity problem (LCP) if the mapping $\Phi(u)$ is a linear function. In recent years, many researchers concentrate on constructing feasible and efficient iteration methods for solving LCP, especially when coefficient A is a positive-definite matrix or an H_+-matrix. For example, when A is a symmetric positive-definite matrix, Dong and Jiang presented a modified modulus method for LCP and gave the optima of the parameter in [5], which was based on the modulus method [6] and combined splitting technique in Bai and Zhang [7], Bai, Yin and Su [8]. Authors Robinson, Feng, Nocedal and Pang studied the two-phase methods for solving both asymmetric and symmetric LCP that consists of an active set prediction stage and an acceleration stage in [9]. By reformulating the LCP as a fixed-point equation and combining the modulus method [6], Bai [10] established several modulus-based matrix splitting iteration methods for LCP when the system matrix A is a positive-definite matrix or an H_+-matrix, which is more practical than the other existing modulus methods, such as the modified modulus method [5] and the extrapolated modulus method [11]. When system matrix A is an H_+-matrix, Xu and Liu [12] improved convergence theories of modulus-based matrix splitting iteration methods. Li [13] extended the modulus-based matrix splitting iteration method to a general case. Furthermore, Bai and Zhang constructed modulus-based synchronous multisplitting iteration methods in [14], which is effective in practical application.

If $\Phi(u)$ is a general function, however, the problem (1.1) belongs to a class of nonlinear complementarity problems which have different important applications in many fields. Therefore, numerous iteration methods have been proposed and studied for solving NCP. See, e.g., [15–18]. Sun and Zeng in [18] proposed a modified semismooth Newton method for solving the problem (1.1) with an M-matrix A and $\Phi : \mathbb{R}^n \to \mathbb{R}^n$ being a continuously diagonal differentiable function on $\mathbb{R}^n$. Huang and Ma in [19] proposed modulus-based matrix splitting iteration methods for solving a class of weakly nonlinear complementarity problems when system matrix A is a positive definite matrix and the nonlinear part is a Lipschitz continuous function; and they also extended the condition positive diagonal matrix Ω to an invertible nonnegative one in [20]. Ma and Huang in [21] presented accelerated modulus-based matrix splitting iteration methods when A is a positive definite matrix and an H_+-matrix. For solving a class of implicit complementarity problems, Hong and Li [22] also proposed the modulus-based matrix splitting iteration methods. They established convergence theories for certain coefficient matrices A. Later, in [28] they studied