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New Reflection Principles for Maxwell's Equations and Their Applications

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Abstract. Some new reflection principles for Maxwell's equations are first established, which are then applied to derive two novel identifiability results in inverse electromagnetic obstacle scattering problems with polyhedral scatterers.

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1. Introduction

The main goal of this work is to establish some new reflection principles for Maxwell's equations when general mixed perfect and imperfect scatterers are involved, and then to apply the reflection principles to the unique determination of the scatterers in inverse electromagnetic scattering problems by either the electric far field patterns or the magnetic far field patterns.

Consider an impenetrable scatterer $\mathbf{D}$, which is assumed to be a compact domain in $\mathbb{R}^3$ and may consist of finitely many pairwise disjoint bounded polyhedra. Suppose the incident fields are taken to be the normalized time-harmonic electromagnetic plane waves of the form (cf. [10])

$$\mathbf{E}^{\mathbf{i}}(\mathbf{x}) := \frac{\mathrm{i}}{k}\mathrm{curl}\,\mathrm{curl}\, p\, e^{\mathrm{i}k\mathbf{x}\cdot d} = \mathrm{i}k(d\times p)\times d e^{\mathrm{i}k\mathbf{x}\cdot d}, \tag{1.1}$$

$$\mathbf{H}^{\mathbf{i}}(\mathbf{x}) := \mathrm{curl}\, p\, e^{\mathrm{i}k\mathbf{x}\cdot d} = \mathrm{i}k d\times p e^{\mathrm{i}k\mathbf{x}\cdot d}, \tag{1.2}$$

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where $\mathrm{i}=\sqrt{-1}$, and $p\in\mathbb{R}^3$, $k>0$ and $d\in\mathbb{S}^2:=\{\mathbf{x}\in\mathbb{R}^3;|\mathbf{x}|=1\}$ represent respectively a polarization, a wave number and a direction of propagation. Then the associated forward scattering problem is described by the following time-harmonic Maxwell's equations (see [10]):

$$\operatorname{curl}\mathbf{E}-\mathrm{i}k\mathbf{H}=0,\quad \operatorname{curl}\mathbf{H}+\mathrm{i}k\mathbf{E}=0\quad\text{in}\quad \mathbf{G}:=\mathbb{R}^3\backslash\mathbf{D}, \tag{1.3}$$

$$\lim_{|\mathbf{x}|\to\infty}(\mathbf{H}^{\mathbf{s}}\times\mathbf{x}-|\mathbf{x}|\mathbf{E}^{\mathbf{s}})=0, \tag{1.4}$$

where $\mathbf{E}=(E_1,E_2,E_3)$ and $\mathbf{H}=(H_1,H_2,H_3)$ are respectively the total electric and magnetic fields formed by the incident fields $\mathbf{E}^{\mathbf{i}}(\mathbf{x})$, $\mathbf{H}^{\mathbf{i}}(\mathbf{x})$ and the scattered fields $\mathbf{E}^{\mathbf{s}}(\mathbf{x})$ and $\mathbf{H}^{\mathbf{s}}(\mathbf{x})$:

$$\mathbf{E}(\mathbf{x})=\mathbf{E}^{\mathbf{i}}(\mathbf{x})+\mathbf{E}^{\mathbf{s}}(\mathbf{x}),\quad \mathbf{H}(\mathbf{x})=\mathbf{H}^{\mathbf{i}}(\mathbf{x})+\mathbf{H}^{\mathbf{s}}(\mathbf{x}). \tag{1.5}$$

We shall assume that the boundary $\partial\mathbf{D}$ of the scatterer $\mathbf{D}$ has a Lipschitz dissection, i.e., $\partial\mathbf{D}=\Gamma_D\cup\Sigma\cup\Gamma_I$, where Γ_D and Γ_I are two disjoint and relatively open subsets of $\partial\mathbf{D}$, having Σ as their common boundary (see [21]). Then we shall further complement the system (1.3)-(1.5) with the following general mixed boundary condition:

$$\nu\times\mathbf{E}=0\qquad\text{on}\quad\Gamma_D, \tag{1.6}$$

$$\nu\times\operatorname{curl}\mathbf{E}-\mathrm{i}\lambda(\nu\times\mathbf{E})\times\nu=0\qquad\text{on}\quad\Gamma_I, \tag{1.7}$$

where ν is the unit outward normal to $\partial\mathbf{D}$, and $\lambda\in C^{0,\alpha}(\Gamma_I)$ is a non-negative Hölder continuous function, with $0<\alpha<1$. Scattering problems with the mixed boundary conditions (1.6)-(1.7) are widely encountered in military and engineering applications. For instance, in order to avoid the detection by radar, hostile objects may be partially coated by some special material designed to reduce the radar cross section of the scattered wave. Boundary conditions (1.6)-(1.7) correspond to the case where the perfect conductor $\mathbf{D}$ is partially coated on the part $\mathbf{\Gamma}_I$ of its boundary with a dielectric. We refer to [3], [4] and [5] for the physical relevance and practical implications of the electromagnetic scattering problems in this setting.

It is known that the forward scattering system (1.3)-(1.7) has a unique solution $(\mathbf{E},\mathbf{H})\in H_{loc}(\mathrm{curl};\mathbf{G})\times H_{loc}(\mathrm{curl};\mathbf{G})$ (see [4] and [7]). And the singular behavior of the weak solution occurs only around the corners and edges, that is, $(\mathbf{E},\mathbf{H})$ satisfies (1.3) in the classical sense in any subdomain of $\mathbf{G}$, which does not meet any corner or edge of $\mathbf{D}$ (see [15]). By the regularity of the strong solution for the forward scattering problem (see [9] and [10]), we know that both $\mathbf{E}$ and $\mathbf{H}$ are $C^{0,\alpha}$-continuous up to the regular points, namely, points lying in the interior of the open faces of $\mathbf{D}$. Moreover, $\mathbf{E}$ and $\mathbf{H}$ are analytic in $\mathbf{G}$ and the asymptotic behavior of the scattered fields $\mathbf{E}^{\mathbf{s}}$ and $\mathbf{H}^{\mathbf{s}}$ is governed by (see [10])

$$\mathbf{E}^{\mathbf{s}}(\mathbf{x};\mathbf{D},p,k,d)=\frac{e^{\mathrm{i}k|\mathbf{x}|}}{|\mathbf{x}|}\left\{\mathbf{E}_\infty(\hat{\mathbf{x}};\mathbf{D},p,k,d)+\mathcal{O}\left(\frac{1}{|\mathbf{x}|}\right)\right\}\quad\text{as}\quad|\mathbf{x}|\to\infty, \tag{1.8}$$

$$\mathbf{H}^{\mathbf{s}}(\mathbf{x};\mathbf{D},p,k,d)=\frac{e^{\mathrm{i}k|\mathbf{x}|}}{|\mathbf{x}|}\left\{\mathbf{H}_\infty(\hat{\mathbf{x}};\mathbf{D},p,k,d)+\mathcal{O}\left(\frac{1}{|\mathbf{x}|}\right)\right\}\quad\text{as}\quad|\mathbf{x}|\to\infty, \tag{1.9}$$