Numer. Math. Theor. Meth. Appl. Vol. **2**, No. 3, pp. 301-325
doi: 10.4208/nmtma.2009.m8016 August 2009

An Explicit-Implicit Predictor-Corrector Domain Decomposition Method for Time Dependent Multi-Dimensional Convection Diffusion Equations

Liyong Zhu[1,2], Guangwei Yuan[1] and Qiang Du[3,*]

[1] *Laboratory of Computational Physics, Institute of Applied Physics and Computational Mathematics, Beijing, China.*
[2] *Division of Mathematical Sciences, School of Physical and Mathematical Sciences, Nanyang Technological University, Singapore, 637371.*
[3] *Department of Mathematics, Pennsylvania State University, University Park, PA 16802, USA.*

Received 3 October 2008; Accepted (in revised version) 16 March 2009

Abstract. The numerical solution of large scale multi-dimensional convection diffusion equations often requires efficient parallel algorithms. In this work, we consider the extension of a recently proposed non-overlapping domain decomposition method for two dimensional time dependent convection diffusion equations with variable coefficients. By combining predictor-corrector technique, modified upwind differences with explicit-implicit coupling, the method under consideration provides intrinsic parallelism while maintaining good stability and accuracy. Moreover, for multi-dimensional problems, the method can be readily implemented on a multi-processor system and does not have the limitation on the choice of subdomains required by some other similar predictor-corrector or stabilized schemes. These properties of the method are demonstrated in this work through both rigorous mathematical analysis and numerical experiments.

AMS subject classifications: 65N30, 65F10
Key words: Convection diffusion equation, parallel algorithm, domain decomposition, modified upwind differences, predictor-corrector, explicit-implicit scheme, convergence analysis.

1. Introduction

Large scale time dependent multi-dimensional convection-diffusion partial differential equations are often used to model many important physical problems. Numerical solutions of these equations are computationally demanding due to the needs to achieve high accuracy and overcome numerical instabilities and stiffness [3, 4, 8]. Solutions on multi-processor computer systems are sometimes the only viable approach for conducting realistic simulation in practice. There are thus considerable interests in developing efficient

*Corresponding author. *Email addresses:* zhu_liyong@iapcm.ac.cn (L. Zhu), Yuan_guangwei@iapcm.ac.cn (G. W. Yuan), qdu@math.psu.edu (Q. Du)

parallel schemes for convection-diffusion problems which share nice stability and accuracy properties.

To help parallelize large scale simulations, domain decomposition can be one powerful tool which has been successfully applied not only to solve many time-independent equations but also to simulate transient phenomena and evolution equations. For time-dependent parabolic problems, there are naturally built decomposition schemes to utilize the time history. For example, the explicit-implicit domain decomposition (EIDD) methods [1, 2, 7] have been studied for many years. In the EIDD algorithm proposed in [1], the values at inter-boundaries may be calculated by explicit schemes with coarser spatial grid size, while those in subdomains are obtained by implicit computation with the finer grid. Such EIDD methods are globally non-iterative, non-overlapping and are computationally and communicationally efficient for each simulation time step. Due to high parallelism and good stability, the studies on EIDD type algorithms have attracted the interests of many researchers. In [28], by adding a stabilization step to EIDD, some stabilized explicit-implicit domain decomposition (SEIDD) methods for the numerical solution of parabolic equations are proposed. The SEIDD methods retain the time-stepwise efficiency in computation and communication of the EIDD methods while maintaining numerical stability. However, flexibility in domain partitioning has to be sacrificed to some extent due to the non-crossover assumption of interior boundaries. In addition, there is no mathematical proof of the improved stability associated with the SEIDD methods so far, though there have been convincing numerical experiments conducted for a wide range of multidimensional parabolic problems. As a further improvement, in [13], a new class of corrected explicit-implicit domain decomposition (CEIDD) methods is presented. Based on non-crossover and crossover types of zigzag interfaces, the resulting CEIDD-ZI algorithms are shown to be convergent in the discrete H^1 semi-norm and L^2 norm. While the CEIDD-ZI scheme allows crossover interior boundaries, the assumption on zigzag interfaces adds complication to the practical implementation and limits the flexibility of domain partitioning. Later on, some new corrected explicit-implicit algorithms have been considered in [11, 12, 17, 18] which, instead of predicting the values at inter-boundary with explicit schemes like that in [13, 28], employ some linear combination of the values on previous two time levels as the predicted values at interior boundary. Some theoretical analysis has also been given in [12].

Recently, we have extended the ideas of [11, 12, 17, 18] to time dependent convection-diffusion equations [27]. In the past, upwind schemes have been widely used in the numerical simulations of such equations. A standard upwind scheme can often avoid numerical oscillations, but it can only get the first-order accuracy. Among many possible improvements, modified upwind difference methods have been used, for example, in the context of explicit-implicit schemes, some works can be found in [19, 20] where methods with relaxed CFL stability conditions were considered. In our recent work, combining the new corrected explicit-implicit domain decomposition method proposed in [11] with the modified upwind differences proposed in [9], a new Explicit-Implicit Predictor-Corrector Modified-Upwind (EIPCMU) scheme has been developed. Yet the true advantage of such an algorithm lies in its multidimensional implementation, especially in terms of allowing very flexible domain decompositions. We therefore present in this paper the extension or